Sparsity-Based Restoration of SMOS Images in the Presence of Outliers

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1. Abstract

- SMOS Mission
  - Soil moisture and ocean surface salinity are of significant importance to improve meteorological and climate prediction
  - The SMOS mission monitor these quantities, by measuring the brightness temperature by means of L-band aperture synthesis interferometry
  - Despite the L-band being reserved for Earth and space exploration, SMOS images reveal large number of strong outliers, produced by illegal antennas emitting in this band.
- Our contribution
  - In this work we propose a variational approach to recover a super-resolved, denoised brightness temperature map by modeling the image formation as the superposition of three super-resolved components in the spatial domain: the target brightness temperature map u, an image o modeling the outliers, and Gaussian noise n.
  - The proposed model is interesting in itself, as it is general enough to be applied to other restoration problems. Experiments on real and synthetic data confirm the suitability of the proposed approach.

2. SMOS Mission

- SMOS Mission Objective: Observe soil moisture over the land and salinity over the oceans
- Principles
  - Moisture and salinity affect microwave radiation emitted from the Earth
  - Measure the microwave radiation (brightness temperature T_b)
  - Use L-band microwave radiometer system
  - Major drawback: antenna's size
  - Solution: Interferometry

Interferometry principle

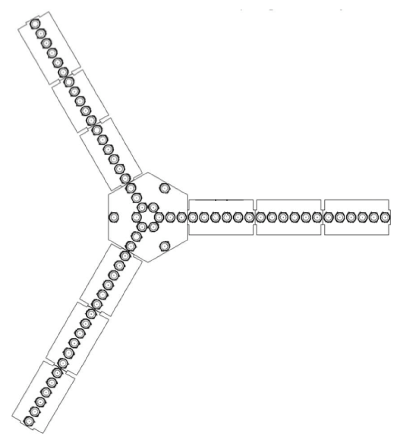
- Measure the phase difference of incident radiation
- Cross-correlation between all pairs of receivers to obtain the *Visibility Function* V_{kl}:

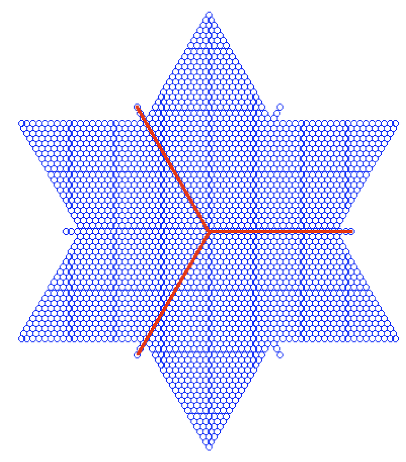
$$V_{k,l} = \frac{1}{\Omega_k \Omega_l} \iint_{||\boldsymbol{\xi}|| \le 1} \frac{U_k(\boldsymbol{\xi}) U_l^*(\boldsymbol{\xi}) \tilde{r}_{kl}(t)}{\sqrt{1 - ||\boldsymbol{\xi}||^2}} (T_b(\boldsymbol{\xi}) - T_r) e^{-i2\pi \mathbf{u}_{kl}^T \boldsymbol{\xi}} d\boldsymbol{\xi}$$

- T_b can be obtained indirectly from V_{kl}

The MIRAS instrument

- Support of T_b is the unit circle
  - Optimum sampling grid on visibilities is an hexagonal grid
  - Two possible configurations: triangular or Y shaped arrays
  - Frequency coverage is larger for Y-shaped (but does not cover the entire hexagonal domain)



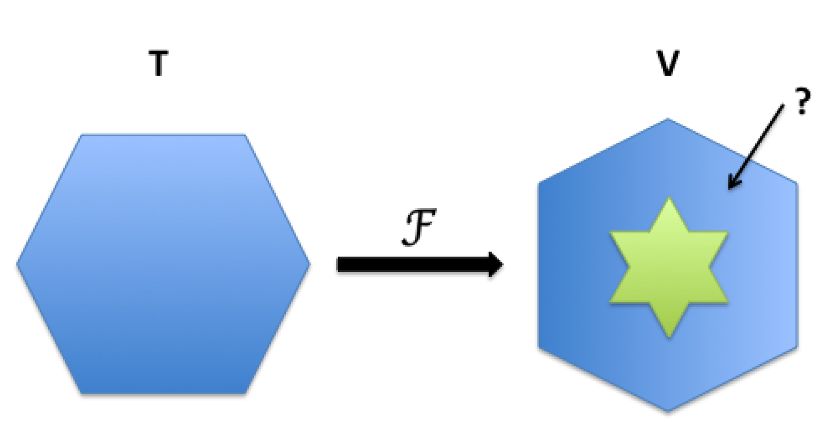


Problem Statement

- If we consider the discrete version of this linear operator, it can be stated by means of matrix $\mathbf{G}$:

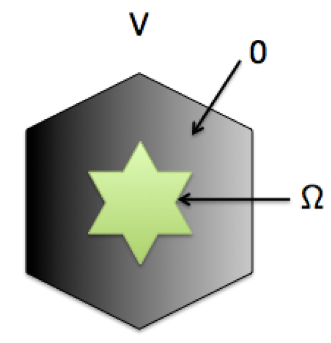
$$\mathbf{G}T = V$$

- $\dim(T) > \dim(V)$: the problem is under constrained: we need to add *a priori* information to regularize it.



- A direct approach: Zero padding regularization (Anterrieu 2004)

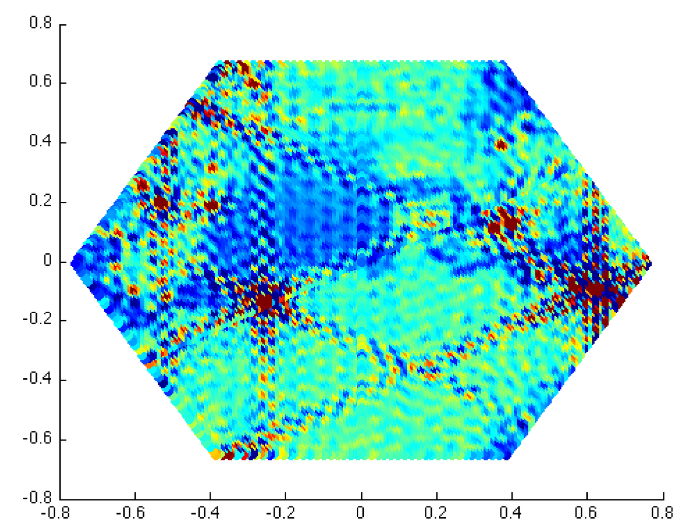
$$\min_T \|V - \mathbf{G}T\|_2^2$$
$$\text{s.t. } (I - P_\Omega)T = 0$$
$$\text{with } P_\Omega = \mathcal{F}^{-1} Z_\Omega Z_\Omega^* \mathcal{F}$$



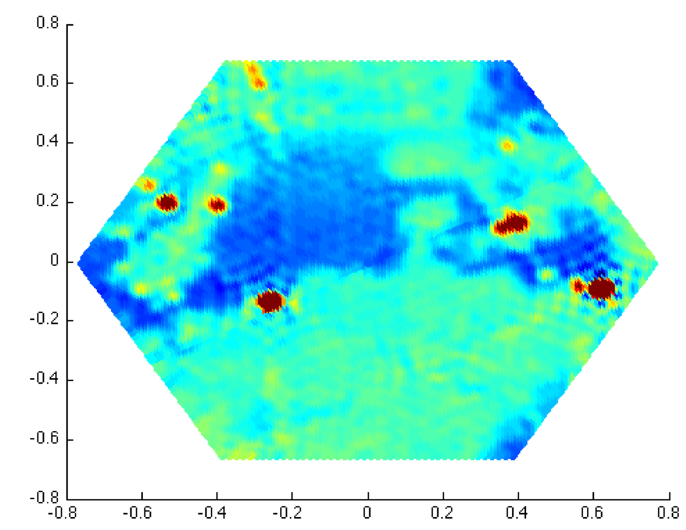
- This problem can be reformulated as:

$$\min_{\hat{T} \in \Omega} \|V - \overbrace{\mathbf{G}\mathcal{F}^{-1} Z_\Omega}^{A} \hat{T}\|_2^2$$

- T can be simply recovered from $\hat{T}$ by $T = \mathcal{F}^{-1} Z_\Omega \hat{T}$
- $\hat{T}$ is SMOS L1B data product (V is SMOS L1A data product)
- Zero padding limitations
  - Outliers: Illegal transmitters that generate strong Gibbs effects
  - Poor spectral extrapolation: limited resolution



Direct Method: $T_b = \mathcal{F}^{-1}(\hat{T})$



Regularized Method - Blackmann: $T_b = \mathcal{F}^{-1}(B\hat{T})$

5. Proposed method

Data Modelization Model the data as the contribution of several original images:

- Image u: Original brightness temperature image of bounded variation: TV semi-norm
- Image o: Outliers image: Sparsity norm (ℓ^1 or ℓ^0)
- Image n: Gaussian image noise: ℓ^2 data fidelity term

Variational Formulation

$$\min_{u,o} \{\mathsf{TV}(u) + \mu S(o)\}$$
$$\text{s.t. } \|W(\mathcal{F}(o+u) - D)\|_2^2 \le |\Omega|\sigma^2$$

where

- D Original L1B data ($\hat{T}$)
- W Weight matrix
- μ Trade-off between sparsity and regularity
- σ is the known noise variance

Unconstrained formulation

$$\min_{u,o} \left\{ \|W(\mathcal{F}(o+u) - D)\|_2^2 + \lambda(\mathsf{TV}(u) + \mu S(o)) \right\}$$

where parameter λ is derived from σ by Uzawa's algorithm

Final Method

$$\min_{u,o} \left\{ \|W(\mathcal{F}(o+u) - D)\|_2^2 + \lambda(\mathsf{TV}(u) + \mu S(o)) \right\}$$
$$\text{s.t. supp } \hat{u} \subseteq \mathcal{H} \subset \mathcal{C}$$

- $\mathcal{H}$: Intermediate cell required to compute spectral TV method (Moisan 2007) $\rightarrow$ reduces staircaising effect

[L. Moisan, How to discretize the total variation of an image? Proc. Appl. Math. Mech, 2007]

Numerical Implementation

Two stage process:

- Stage one Solve the minimization problem with sparsity term $S(o) = \|o\|_1$
  - the problem is convex
  - can be solved iteratively with a Forward-Backward algorithm
  - converges to a global minimum
- Stage two Starting from the previous solution, we solve the same problem with $S(o) = \|o\|_0$
  - the problem is non-convex due to the ℓ_0 norm
  - for this functional the Forward-Backward algorithm converges to a local minimum [Blumensath and Davies 2005]

Fordward-Backward method [Combettes-Wajs 2005]

$$E(x) = \underbrace{\|W(\mathcal{F}(o+u) - D)\|_2^2}_{E_1} + \underbrace{\lambda(\mathsf{TV}(u) + \mu S(o))}_{E_2}$$

Implementation of proximal operators

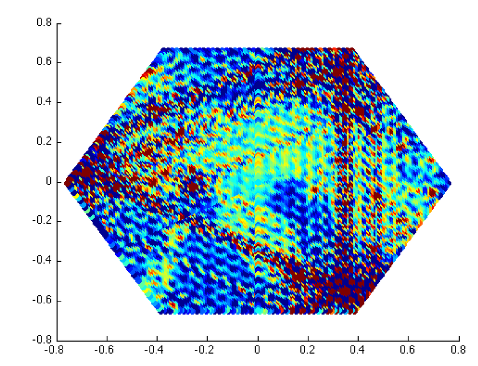
- $\mathsf{prox}_{\gamma E_2}(u, o) = (\mathsf{prox}_{\gamma\lambda\mathsf{TV}}(u), \mathsf{prox}_{\gamma\lambda\mu S}(o))$
- $\mathsf{prox}_{\gamma TV}$: modified version of [Chambolle 2004] with spectral projection
- $\mathsf{prox}_{\gamma\|\cdot\|_1}$: the *soft-threshold* or *shrinkage* operator
- $\mathsf{prox}_{\gamma\|\cdot\|_0}$: the *hard-threshold* operator

Proximal operator

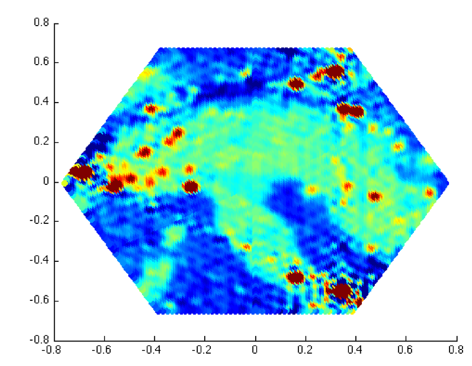
$$\mathsf{prox}_{E_2}(x) = \arg\min_y E_2(y) + \frac{1}{2}\|x - y\|^2$$

Experiments on real data

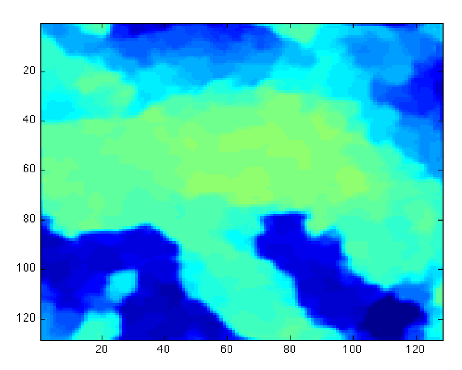
$\mathcal{F}^{-1}$



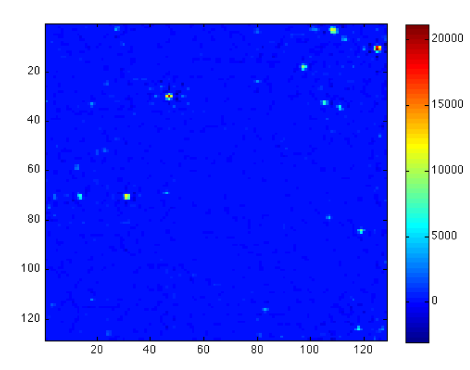
Blackman



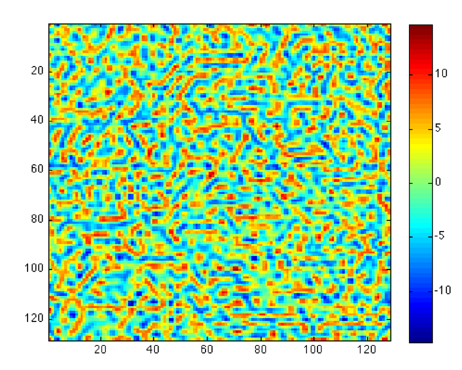
The proposed method



Outliers



Noise



Conclusions

We propose a variational method to restore images from the L1B SMOS data product.

- The method models the observations as the superposition of three components on the spatial domain:
  - The target brightness temperature map u
  - The outliers image o due to the illegal emissions
  - A gaussian noise image n
- The method also extrapolates the spectral domain of u thanks to the total variation semi-norm
- The method is general enough to be used for other restoration problems

Future work

- Modify the method to include directly the raw L1A (visibilities) data
- Improve convergence rate using other optimization algorithms (FISTA, mFISTA)
- Systematic evaluation of the method using real and simulated data provided by CESBIO

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