1. Abstract

• SMOS Mission
  • Soil moisture and ocean surface salinity are of significant importance to improve meteorological and climate prediction
  • The SMOS mission monitors these quantities, by measuring the brightness temperature by means of L-band aperture synthesis interferometry
  • Despite the L-band being reserved for Earth and space exploration, SMOS images reveal large number of strong outliers, produced by illegal antennas emitting in this band.
  • Our contribution
    • In this work we propose a variational approach to recover a super-resolved, denoised brightness temperature map by modeling the image formation as the superposition of three super-resolved components in the spatial domain: the target brightness temperature map \( u \), an image \( o \) modeling the outliers, and Gaussian noise \( \epsilon \).
    • The proposed model is interesting in itself, as it is general enough to be applied to other restoration problems.
    • Experiments on real and synthetic data confirm the suitability of the proposed approach.

2. SMOS Mission

• SMOS Mission Objective: Observe soil moisture over the land and salinity over the oceans
• Principles
  • Moisture and salinity affect microwave radiation emitted from the Earth
  • Use L-band microwave radiometer system
  • Major drawback: antenna’s size
• Interferometry principle
  • Cross-correlation between all pairs of receivers to obtain the Visibility Function \( V_{ij} \): 
    \[ V_{ij} = \frac{1}{2\pi} \int \int \frac{G(\xi)G(\eta)F(u)(\xi,\eta)F(v)(\xi-\xi',\eta-\eta')}{\sqrt{||\xi||^2+||\eta||^2}} \, d\xi \, d\eta \]
  • \( T \) can be obtained indirectly from \( V_{ij} \).
• The MIRAS instrument
  • Support of \( T \) is the unit circle
  • Optimum sampling grid on visibilities is an hexagonal grid
  • Two possible configurations: triangular or Y shaped arrays
  • Frequency coverage is larger for Y-shaped (but does not cover the entire hexagonal domain)

Problem Statement

If we consider the discrete version of this linear operator, it can be stated by means of matrix \( G \):
\[ GT = V \]
\[ \text{dim}(T) > \text{dim}(V) \] the problem is under constrained: we need to add a priori information to regularize it.

• A direct approach: Zero padding regularization (Anterrieu 2004)
  \[ \min_{T} \left\{ \|V - GT\|^2_F \right\} \quad \text{s.t.} \quad (I - F_1)T = 0 \quad \text{with} \quad F_1 = F^* Z_0 Z_1^* F \]
  This problem can be reformulated as:
  \[ \min_{T} \left\{ \|V - GT\|^2_F \right\} \quad \text{s.t.} \quad F_1 T = 0 \]
  \( T \) can be simply recovered from \( F_1 \) by \( T = F^* Z_0 Z_1^* F \)

• Zero padding limitations
  • Outliers: Illegal transmitters that generate strong Gibbs effects
  • Poor spectral extrapolation: limited resolution

5. Proposed method

Data Modeling

• Image \( u \): Original brightness temperature image of bounded variation: \( TV \) semi-norm
• Image \( o \): Outliers image: Sparsity norm \( \|\ell\|_0 \) or \( \ell^1 \)
• Image \( \epsilon \): Gaussian image noise: \( \epsilon^2 \) data fidelity term

Variational Formulation

\[ \min \left\{ TV(u) + \mu S(o) \right\} \quad \text{s.t.} \quad \|W(F(o + u) - D)|^2 \right\|_2 \leq \|\epsilon\|^2 \]

where
- \( D \): Original LIB data \( T \)
- \( W \): Weight matrix

Final Method

\[ \min \left\{ \|W(F(o + u) - D)|^2 + \lambda \mu S(o) \right\} \]

\( \lambda: \) Intermediate cell required to compute spectral TV method (Moisan 2007) \( \rightarrow \) reduces staircasing effect


Numerical Implementation

Two stage process:
- Stage one Solve the minimization problem with sparsity term \( S(o) = \|\ell\|_0 \):
  • the problem is convex
  • can be solved iteratively with a Forward-Backward algorithm
  • converges to a global minimum
- Stage two Starting from the previous solution, we solve the same problem with \( S(o) = \|\ell\|_2 \):
  • the problem is non-convex due to the \( \ell_0 \) norm
  • for this functional the Forward-Backward algorithm converges to a local minimum [Blumensath and Davies 2005]

Fordward-Backward method [Combettes-Wajs 2005]

\[ E(x) = \|W(F(o + u) - D)|^2 + \lambda TV(u) + \mu S(o) \]

Implementation of proximal operators
- \( \text{prox}_{\ell_0} (a) = \{a\}_{a \text{ does not sum up to } 0} \)
- \( \text{prox}_{\ell_1} (a) = \{a\}_{a > 0} \) maxima process
- \( \text{prox}_{\ell_{1/2}} \): soft-thresholding or shrinkage operator
- \( \text{prox}_{\ell_{\infty}} \): the hard-threshold operator

Proximal operator

\[ \text{prox}_{\ell_0}(a) \rightarrow \text{arg min } \frac{1}{2} \|a - b\|^2 + \frac{\lambda}{2} \|a\|_1 \]

Experiments on real data

Conclusions

We propose a variational method to restore images from the LIB SMOS data product.
- The method models the observations as the superposition of three components on the spatial domain:
  • The target brightness temperature map \( u \)
  • The outliers image \( o \) due to the illegal emissions
  • A Gaussian noise image \( \epsilon \)
- The method also extrapolates the spectral domain of \( u \) thanks to the total variation semi-norm
- The method is general enough to be used for other restoration problems

Future work

- Improve convergence rate using other optimization algorithms (FISTA, mFISTA)
- Systematic evaluation of the method using real and simulated data provided by CESBIO