Control strategies of selective harmonic current shunt active filter

G. Casaravilla, A. Salvia, C. Briozzo and E. Watanabe

Abstract: The possible calculation methodologies when designing a selective shunt active filter are presented. To accomplish selective extraction of harmonic sequences, the modulation-filter-demodulation technique is used. The fundamental equations of this method are based on \( pq \) theory. Its equivalence with the SRF (synchronous reference frame) method is shown. In order to validate the proposed calculation methods, measured currents from an arc furnace, showing high harmonic distortion, are used. The obtained results show the effectiveness of the proposed method for selective filtering of the undesired current harmonics in a controlled way.

1 Introduction

In the last few decades, the evolution of two aspects concerning power systems has created conditions for the more extended use of active filters. The first aspect is related to power semiconductor device development. Converters capable of synthesising voltages and currents with an adequate band-width for harmonic current compensation at MVA-level are now available at competitive prices. The other aspect is the gradual application of regulations limiting the generation of harmonic currents by the customers.

Active filters are ideally suited for filtering localised harmonic currents in a guided way, allowing the application of the concept 'you dirty, you clean'. This concept cannot be applied using conventional passive filters. Furthermore, the use of active filters can eliminate some of the problems of passive filters such as poor tuning due to dispersion of their characteristic parameters, and resonances due to the impedance of the surrounding electrical network.

Among the different methods, the use of \( pq \) theory (active and imaginary power) [1], has proved to be especially adequate for controlling active filters. In particular, it has been used for separating the residual harmonics, then eliminating (as theory indicates) or reducing (as in practice) the harmonic distortion. For the control of selective active filters several papers have reported the use of the SRF (synchronous reference frame) method [2-6], which is definitively a particular case of the application of the \( pq \) method using harmonic voltages as references. In this work, the contribution to this topic in [7] is generalised.

In [8], the design of a minimum cost active filter is presented. The optimisation is made by setting the percentage of each harmonic to be eliminated just to meet the regulation requirements. The obtained results could be acceptable, but with the employed control strategy, the reduction of some of the harmonics become quite troublesome. The question of whether better results could be achieved by reducing each harmonic current in a controlled way in order to exactly meet the regulation requirements arises naturally. One answer to this question can be obtained from the results of using selective filters.

2 \( pq \) theory, harmonic and sequence contributions to \( p \) and \( q \) spectrum

\( pq \) theory [1] is basically a time domain analysis tool. In a stationary periodic process it is possible to do a frequency domain analysis. Current and voltage harmonics and sequences appear explicitly. The following equations are explained in detail in [7, 9, 10]:

\[
v_k(t) = \sum_{n=1}^{\infty} \sqrt{2} V_{kn} \sin (wn t + \phi_{kn})
\]

\[
i_k(t) = \sum_{n=1}^{\infty} \sqrt{2} I_{kn} \sin (wn t + \delta_{kn})
\]

where \( V_{kn} \) and \( I_{kn} \) are \( k \)-phase, \( n \)-harmonic voltage and current amplitudes, and \( \phi_{kn} \) and \( \delta_{kn} \) their arguments.

\[
\begin{bmatrix}
\frac{I_m}{I_m} \\
\frac{I_n}{I_m}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2
\end{bmatrix} \begin{bmatrix}
\frac{I_m}{I_m} \\
\frac{I_m}{I_m}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{I_m}{I_m} \\
\frac{I_m}{I_m}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
\frac{I_m}{I_m} \\
\frac{I_m}{I_m}
\end{bmatrix}
\]

\[
i_{on} = \sqrt{2} I_{on} \sin (w_0 t + \delta_{on}) + \sqrt{2} I_{on} \sin (w_0 t + \delta_{on}) + \sqrt{2} I_{on} \sin (w_0 t - \delta_{on})
\]

\[
i_{on} = \sqrt{2} I_{on} \sin (w_0 t + \delta_{on}) + \sqrt{2} I_{on} \sin (w_0 t + \delta_{on}) + \sqrt{2} I_{on} \sin (w_0 t + \delta_{on})
\]

\[
i_{on} = \sqrt{2} I_{on} \sin \left( w_0 t + \frac{2 \pi}{3} \right) + \sqrt{2} I_{on} \sin \left( w_0 t + \frac{2 \pi}{3} \right) + \sqrt{2} I_{on} \sin \left( w_0 t + \frac{2 \pi}{3} \right)
\]
where \( I_{r+} = I_{r+}(\delta_{r+}) \) and \( I_{r-} = I_{r-}(\delta_{r-}) \).

The current and voltage of a, b, and c phases are given in (1), (2), and (3) are the direct and inverse calculation of these harmonics in positive, negative and zero sequences. The result is (4), where harmonics and sequences are present. Equation (5) represents the transform from three to two coordinates (Clarke transform) resulting in (6), where those components are expressed in the reference axes \( u \) and \( p \) in terms of all the harmonic sequence components. Similar expressions can be obtained for the voltages.

\[
[i_s] = \sqrt{2/3} \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2 
\end{bmatrix} [i_{r+}],
\]

(5)

\[
ix(t) = + \sum_{n=1}^{\infty} \sqrt{3} I_{r+} \sin(w_nt + \delta_{r+}) + \sum_{n=1}^{\infty} \sqrt{3} I_{r-} \sin(w_nt + \delta_{r-})
\]

(6)

\[
ib(t) = + \sum_{n=1}^{\infty} \sqrt{3} I_{r+} \cos(w_nt + \delta_{r+}) + \sum_{n=1}^{\infty} \sqrt{3} I_{r-} \cos(w_nt + \delta_{r-})
\]

The definition of instantaneous real power \( p \) and imaginary power \( q \) [1], is the one shown in (7) with its decomposition in \( \dot{p} \) (\( p \) mean value) and \( \dot{p} \) (\( p \) ripple) as shown in (9) and (10). The table in Fig. 1 shows how these sequences are finally placed in the \( p \) and \( q \) frequency spectrum.

\[
\begin{bmatrix}
p & q \\
ix & ib
\end{bmatrix}
= \begin{bmatrix}
vx & v\beta \\
v\beta & -vx
\end{bmatrix}
\begin{bmatrix}
ix & ib
\end{bmatrix}
\]

(7)

\[
ix = \frac{1}{\sqrt{2}} \begin{bmatrix} vx \\ v\beta \end{bmatrix}, \quad ib = \begin{bmatrix} v\beta \\ -vx \end{bmatrix}
\]

(8)

\[
p(t) = + \sum_{n=1}^{\infty} \sqrt{3} V_{r+} I_{r+} \cos((\phi_{r+} - \delta_{r+}) t)
\]

(9)

\[
q(t) = + \sum_{n=1}^{\infty} \sqrt{3} V_{r-} I_{r-} \cos((\phi_{r-} - \delta_{r-}) t)
\]

(10)

3 Shunt selective active filter

A shunt selective filter connected as in Fig. 2, must be able to determine, in real time, the current to be taken by the active filter in order to eliminate a certain harmonic sequence from the source system. The voltage used in the calculations is supposed to be only a positive sequence voltage of frequency \( w_c \). Thus, in this case, the corresponding voltage expression for (6) indicates that the voltage is given by (11), where the respective amplitude and phase were eliminated. The \( ^{\prime} + ^{\prime} \) sign indicates that a positive sequence is used. Then, using the \( p \) and \( q \) definitions from (7), (12) is obtained.

\[
\begin{bmatrix}
p & q \\
ix(t) & ib(t)
\end{bmatrix}
= \begin{bmatrix}
+ \sin(w_c t) & -\cos(w_c t) \\
-\cos(w_c t) & + \sin(w_c t)
\end{bmatrix}
\]

(11)

\[
\begin{bmatrix}
p(t) & q(t) \\
ix(t) & ib(t)
\end{bmatrix}
= \begin{bmatrix}
+ \sin(w_c t) & -\cos(w_c t) \\
-\cos(w_c t) & + \sin(w_c t)
\end{bmatrix}
\begin{bmatrix}
ix(t) & ib(t)
\end{bmatrix}
\]

(12)

At this point the similarity between this calculation method and the SRF method [2, 4, 11] should be remarked. In the SRF method, the currents \( i_x \) and \( i_b \) are decomposed in the \( d \) and \( q \) synchronous reference frame of the voltage sequence associated with \( w_c \).

That transformation (rotation) of the axes \( x \) and \( b \) to \( d \) and \( q \) is shown in (13). This expression can also be written as (14). Then, comparing the expressions in (12) and (14), it can be established that except for the change in the sign of \( q \), the SRF method and the \( pq \) theory method developed here are similar [4]. Furthermore, if the amplitude and phase restrictions imposed in (11) were eliminated, the meanings of \( p \) and \( q \) would no longer be the conventional ones. It can be noticed that they could be interchanged in the case that the real phase differs by 90° from the

Fig. 1 Harmonic sequences in \( p \) and \( q \) spectrum. For example, due to \( I_2 \) and \( V_1 \) there is third harmonic in \( p \) or \( q \), and due to \( I_4 \) and \( V_4 \) there is seventh harmonic in \( p \) or \( q \)

Fig. 2 Shunt active filter
Fig. 3  Modulation, selective filtering and demodulation

calculated one.

\[
\begin{align*}
\alpha(t) &= \left[ + \cos(w_c t) - \sin(w_c t) \right] + \left[ + \sin(w_c t) + \cos(w_c t) \right] i(t) \\
\beta(t) &= \left[ - \sin\left(\frac{\pi}{3} - w_c t\right) - \cos\left(\frac{\pi}{3} - w_c t\right) \right] + \left[ - \cos\left(\frac{\pi}{3} - w_c t\right) + \sin\left(\frac{\pi}{3} - w_c t\right) \right] i(t) \\
\alpha(t) &= \left[ + \sin\left(\frac{\pi}{3} - w_c t\right) - \cos\left(\frac{\pi}{3} - w_c t\right) \right] + \left[ + \cos\left(\frac{\pi}{3} - w_c t\right) + \sin\left(\frac{\pi}{3} - w_c t\right) \right] i(t)
\end{align*}
\]

Looking at Fig. 1, it may be seen that the \( p \) and \( q \) spectrum that results from the calculations in (12) presents at zero frequency (DC), and only the current sequence which is wanted is identified and separated.

If this DC portion of the spectrum is separated with a low pass filter and then the inverse calculation (7), which uses the same reference voltage, is applied, compensation currents \( i\alpha_F \) and \( i\beta_F \) are obtained. This operation is represented in the diagram of Fig. 3. The first stage performs a modulation, the intermediate stage is a filter and the last stage performs a demodulation [4, 7]. The multiplication of both channels by \(-1\) after the filters \( G_1(w) \) and \( G_2(w) \) is associated with the current sign convention shown in Fig. 4. The main idea is that the required \( p \) and \( q \), given by the calculations, should be supplied by the active filter. They are then eliminated from the line current \( I \).

Equation (15) is the same as (12), but in the frequency domain, where * denotes convolution and \( S_{uc} \) and \( C_{uc} \) are defined in (16).

\[
\sin(w_c t) \Rightarrow S_{uc}(w) = \frac{1}{2\pi} \left[ \delta(w - w_c) - \delta(w + w_c) \right] \\
\cos(w_c t) \Rightarrow C_{uc}(w) = \frac{1}{2\pi} \left[ \delta(w - w_c) - \delta(w + w_c) \right]
\]

Afterwards, in order to obtain the magnitudes \( p_F \) and \( q_F \) that should be demodulated (Fig. 3), (17) can be written, where the multiplier \(-1\) is also included. From (7), the expression in (18) can be written. In the frequency domain,

\[
\left[ \begin{array}{c}
I\alpha(w) \\
I\beta(w)
\end{array} \right] = \frac{1}{2\pi} \left[ \begin{array}{cc}
+ S_{uc}(w) - C_{uc}(w) \\
- C_{uc}(w) - S_{uc}(w)
\end{array} \right] \left[ \begin{array}{c}
P_F(w) \\
Q_F(w)
\end{array} \right] \\
\Rightarrow I\alpha_F(w) = \frac{1}{2\pi} \left[ \begin{array}{cc}
+ S_{uc}(w) & - C_{uc}(w) \\
- C_{uc}(w) & - S_{uc}(w)
\end{array} \right] \left[ \begin{array}{c}
P_F(w) \\
Q_F(w)
\end{array} \right]
\]

(18) is transformed into (19). Replacing (15) and (17) in (19), (20) is obtained after several calculations. The notation is defined in (21).

\[
\left[ \begin{array}{c}
P_F(w) \\
Q_F(w)
\end{array} \right] = \left[ \begin{array}{cc}
G_1(w) & 0 \\
0 & G_2(w)
\end{array} \right] \left[ \begin{array}{c}
P(w) \\
Q(w)
\end{array} \right] \\
\Rightarrow I\alpha_F(t) = \frac{1}{2\pi} \left[ \begin{array}{cc}
+ S_{uc}(w) & - C_{uc}(w) \\
- C_{uc}(w) & - S_{uc}(w)
\end{array} \right] \left[ \begin{array}{c}
P_F(t) \\
Q_F(t)
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
P_F(w) \\
Q_F(w)
\end{array} \right] = \left[ \begin{array}{cc}
1 & 0 \\
0 & 1
\end{array} \right] \left[ \begin{array}{c}
P_F(w) \\
Q_F(w)
\end{array} \right]
\]

(20)

Looking at the multipliers of \( I\alpha \) and \( I\beta \) in Fig. 5, it should be noted that the possible results are placed in the frequencies \(+w_c\) and \(-w_c\). Hence, only the positive and negative sequences associated with frequency \( w_c \) shown in (25) will be taken into consideration. Any other frequency

Fig. 4  Sign convention

Fig. 5  G, \( G^{-1} \) and \( G^{-1} \) ideal filters
will not appear in the output $I_{ZF}$ and $f_{ZF}$.

$$I_{ZF}(w) = -\frac{1}{4} \left( -jH^2 + H^{-1} \right) H^2 + H^{-1}$$

$$f_{ZF}(w) = -\frac{1}{4} \left( -jH^2 - H^{-1} \right) H^2 - H^{-1}$$

$$I_{ZF}(w) = -\frac{1}{2} \left[ G^2 + G^{-1} \right] I_x + j(G^{-1} - G^1) f_{ZF}$$

$$f_{ZF}(w) = -\frac{1}{2} \left[ j(G^{-1} - G^1) \right] I_x + (G^1 + G^{-1}) f_{ZF}$$

At first it will be seen what happens to the positive sequence shown in (25). Using the Fourier transform property of a delayed-time signal (26), the terms of interest can be located shown in Fig. 3 with a positive sequence frequency only has the positive sequence current associated with $w$.

The result in (28) is obtained from the same reasoning and calculations for the negative sequence associated with $w$.

As a partial conclusion it can be said that as a consequence of the modulating-low pass filter-demodulating procedure of Fig. 3 with a positive sequence $+w$, the obtained output current only has the positive sequence current associated with $w$. In the same way, it can be demonstrated that the same result is obtained if a negative sequence of frequency $-w$ is used for modulating and demodulating (it is enough to substitute $w$ for $-w$). This important result, partially shown in [7], is the same as the result which has been obtained with the SRF methodology, as has already been mentioned.

### 4 Control strategies

As shown in [12], there are three ways of controlling a shunt active filter: measuring the load current, measuring the line current or measuring the voltage at the point of common connection (PCC) of the load. A study of the second case will be made in this work, but the generalisation for the other two cases can be easily done.

With the sign convention of Fig. 4, the system transference between line current $i_L$ and load current $i_C$ is shown in Fig. 7. This scheme can also be outlined as Fig. 8, named here as a selective filter basic cell (SFBC), where it is assumed that the input is the load current $i_C$, and the outputs are the filter current $i_F$ and the line current $i_L$. The parameter $\lambda$ introduced in Fig. 8 indicates the continuous (DC) gain of the low pass filter $G(w)$. It should be noted that if $\lambda$ varies in the interval $[1, 0]$, the filtered harmonic content of the load current varies from 100% to 0. This feature is very useful when designing a minimum size filter that satisfies certain harmonic distortion requirements [8].

If the currents $i_L$ and $i_F$ are modulated and demodulated with a positive sequence, and the inverse transform of (5) is applied to (20), the resulting transference is $i_L(w) = i_C(w) = -G_C(w)$, where $G_C(w) = G(w + w_C)$. In the case of modulating and demodulating with a negative sequence, the definition of $G_C(w)$ is $G_C(w) = G(w + w_C)$. The final transference of the basic cell of Fig. 8 is $i_C(w) = [1 - G_C(w)]$.

It has been shown that by means of the calculations established in Fig. 7, and outlined in Fig. 8 with a SFBC, it is possible to identify and separate a certain harmonic sequence. As a consequence, the current associated with several harmonic sequences that are required to be eliminated from the line current $i_L$ can be calculated in at least two primary ways. The calculation methods in Figs. 9 and 10 will be referred as series (S) and parallel (P). Both calculation methods would arrive at the same result if the modulation-demodulation filters $G_C(w)$ were ideal.

In practice, this does not happen; although the P method appears faster because of the possibility of making calculations in parallel, the S method gives better practical results. Analysing the calculation methods with an ideal filter $G_{CR}$, both methods arrive at the same transference

$$i_L = i_C + \sum (1 - \lambda) i_L + i_C$$

On the other hand, with real filters $G_{CR}$, in the series S case the transference is

$$\frac{i_L}{i_C} = \prod (1 - G_{CR})$$

![Figure 6](image1)

**Figure 6** Graphic operation of (24)

![Figure 7](image2)

**Figure 7** Control system block diagram
Fig. 8 Selective filter basic cell (SFBC)

\[ i'c = i + \frac{1}{(1 - \lambda_2 \lambda_m)} \sum_{i=2}^{m} i \lambda_i \]

Fig. 9 S method, where \( i_1, i_2, ..., i_m \) represent current sequences of \( 1, ..., m \) harmonics \( i \) represents non-filtered harmonic sequences and \( \lambda_1, \lambda_2, ..., \lambda_m \) are SFBC ideal low pass filters gains

\[ i_L = i + \sum_{i=2}^{m} i \lambda_i \]

\[ i = i_1 + \sum_{i=2}^{m} i \lambda_i \]

\[ i_2 = i_2 + \sum_{i=3}^{m} i \lambda_i \]

\[ i_3 = i_3 + \sum_{i=4}^{m} i \lambda_i \]

\[ i_m = i_m + \sum_{i=m+1}^{m} i \lambda_i \]

Fig. 10 P method, where \( i_1, i_2, ..., i_m \) represent current sequences of \( 1, ..., m \) harmonics \( i \) represents non-filtered harmonic sequences and \( \lambda_1, \lambda_2, ..., \lambda_m \) are SFBC ideal low pass filters gains

\[ i_L = i + \sum_{i=2}^{m} i \lambda_i \]

This P method has a great deal of interference between the different selective filters, and is therefore inapplicable. On the other hand, if the line feeder current \( i_L \) is measured, the results are symmetric: the P method is better than the S method, instead of S being better than P as in the C method.

5 Results

To evaluate the real possibilities of filtering several harmonic sequences simultaneously, a real application in an arc furnace [8] is taken as an example. This case is more difficult to deal with because the current to be filtered is not at steady state. The goal is to design a selective filter that could attenuate definite harmonic sequences to a pre-calculated value, in order to obtain a given total harmonic distortion and a given amplitude for each individual harmonic not exceeding the penalisation limits established in the regulation [13]. The filtered sequence components were 18 \((+2, -2, +3, -3, ..., +10, -10)\). Digital Butterworth filters \( G(s) \) were used as low pass filters. To obtain an optimal filtering, the order and the cut-off frequency of the filters were calculated for each harmonic. Fig. 11 shows the ability of the performed calculations to individually synthesise the harmonic currents to be synthesised by the active filter for controlling the total and individual distortion.

Fig. 11 Method S with all \( \lambda_i = 1 \)

One consequence of doing the calculation harmonic by harmonic is that each basic cell takes small portions of the fundamental current; hence the first harmonic current taken by the filter after 18 calculations is unacceptable. To solve this problem, the first harmonic current must be eliminated from the filter current by means of a filtering cell as shown in Fig. 12. In this last filtering cell the filter \( G(s) \) is now a high-pass filter, in what some authors call a direct harmonic elimination method [4, 7, 11], and is presented in [1] as a way of removing sequence +1 and filtering the harmonic residue. These simulation results are worse than those reported in [8], but the searched selectivity gives some benefits. Then, the result shown in Fig. 13 is obtained if the values \( \lambda_i \) are calculated in order to attenuate the harmonics in an individually controlled way, combining the criterion of following the regulations with the criterion of having the minimum total current in the active filter (these calculations will be reported in a future work). This Figure shows the ability of the performed calculations to individually synthesise the harmonic currents to be synthesised by the active filter for controlling the total and individual distortion. Fig. 14 shows the original load current, the actual obtained line current and the active filter current for the optimum \( \lambda_i \). The 12% desired final total distortion is achieved.

Taking as a reference the calculation time indicated in [6], the calculation time used for compensating those 18 harmonics in an individual and selective way, is about 0.18 ms per sample. Thus, the sample and modulation frequency of the active filter inverter will be 5 kHz.
equivalence with the SRF method has been presented. Two calculation alternatives, series (S) or parallel (P), have been presented. If the compensation is made by measuring the load current \( i_L \), the performance of the S method is better because less interference between the filters of different harmonic sequences occurs. If we feedback the line current \( i_L \), symmetric results are introduced, so the P method results are better than those of the S method.

As a way of testing the methodology, real non-periodic waveform currents with a great harmonic distortion were used, and the results show that selective filtering can be achieved.

A minimum cost active filter, which also meets the applicable regulations requirements, can be designed using this selective calculation methodology.

6 Conclusion

The theory associated with the way of calculating the selective shunt active filter current using pq theory and its

7 References

8. CASARAVILLA, G.; BRISSOZZO, C.; and WATANABE, E.: 'Filtro ativo de mínimo custo ajustado a carga de um horno de arco y a las regulamentaciones sobre emisión armónica aplicables'. XII CBA-Congresso Brasileiro de Automaática, 2000, pp. 1108-1113