DISTRIBUTION NETWORK LOSS ALLOCATIONS WITH DISTRIBUTED GENERATION USING NODAL PRICES

J. Mario Vignolo
Instituto de Ingeniería Eléctrica
Universidad de la República
Montevideo, Uruguay
email: jesus@fing.edu.uy

Paul M. Sotkiewicz
Public Utility Research Center
Department of Economics
University of Florida
Gainesville, Florida USA
email: paul.sotkiewicz@cba.ufl.edu

ABSTRACT
We propose employing nodal factor pricing, a method associated with allocating losses at EHV transmission levels, for the allocation of loss costs at the distribution level. This method differs from traditional methods of averaging losses across customers regardless of location, time of use, or the marginal contribution of net power injection/withdrawal positions to losses. With respect to distributed generation (DG) resources, nodal prices provide more efficient price signals for dispatch and siting decisions. Moreover, nodal prices provide greater economic incentives for the deployment of DG by rewarding DG resources for contributions toward reducing losses at the margin through changed power flows. Nodal pricing factors are calculated using power flows locating "the reference bus" at the power supply point where the transmission network connects to the distribution network. We assume no network constraints at the distribution level. Finally, we conclude with an application of this method in a rural radial distribution network.

KEY WORDS
Distribution Networks, Distributed Generation, Loss Allocations

1 Introduction

Over the last decade, there has been an increased interest in distributed generation resources (DG), defined as generation that is directly connected into the distribution network instead of the transmission network, both from governments and researchers, as DG seems to have the potential to change the current structure of power systems. The Working Group 37.23 of CIGRE has summarized in [9] some of the reasons for an increasing share of DG in different countries. Further considerations about the definition of DG can be found in [8].

It is widely accepted (and can be found in several publications) that DG can provide benefits to the network; e.g., reducing losses, acting as a network service provider (i.e. postponing new distribution reinforcements) and providing ancillary services. In addition, being a modular technology it may present a lower cost addition to the system in that a big facility need not to be built that has excess capacity for some years.

Consequently, we are interested in modelling the distribution network with DG to examine the incentive to deploy DG under different tariff structures for the allocation of loss costs. The cost of losses is an operational cost to be recovered by the distribution company (Disco). Different cost allocation methodologies will have different financial impacts on network users, especially DG. For instance, the averaging of losses across space and time among all customers, which is done in many regulatory paradigms, eliminates the price signals that would recognize all of the benefits of load locating close to the transmission/distribution interface or of installing DG, which may reduce the amount of losses, and by extension, the cost of operating the system. Consequently, loss averaging cost allocations may not provide sufficient incentives for the entry of new DG.

As mentioned in [1] several schemes have been proposed for evaluating and pricing line losses since the advent of competitive electricity markets. Many of the schemes remain unsatisfactory in that they either do not provide economically efficient signals and/or the rely on poor approximations. For example in [2], and similarly in [3], a basic assumption of proportionality is introduced to determine the proportion of each generators' to the active power flow in a transmission line. This proportion of line determines the loss allocation rather than an allocation based on actual injections or withdrawals, thereby rendering the price signals on losses inefficient. A loss allocation schemes for multilateral trades based on a quadratic approximation of losses are proposed in [4] and [5]. However, development of the loss allocation formula in [5] is predicated on several approximations, leading to significant differences between losses calculated from the AC power flow solution and those obtained from the proposed scheme ($\approx$15 %). While the method recognizes the impact of counter flows, it does not provide appropriate signals to users of the network to motivate economically efficient operating decisions.

In [1] the marginal loss coefficients (MLCs) are used to allocate losses where the marginal loss coefficients measure the change in total active power losses due to a marginal change in consumption/generation of active power and reactive power at each node in the network.

Because the marginal losses reflect the actual short-run marginal costs they are economically efficient price signals in the short term. Total losses collected equal the summation of the MLCs of each node multiplied by the corresponding consumption/generation of active and reactive power, and are approximately twice the amount of losses due the approximately quadratic relationship between losses and power flow. Because many tariff regimes do not allow for the overcollection of losses, the authors present two reconciliation methods that will ensure that there is no overcollection of losses, but at the expense of dampening the price signals.

In this paper we present a method that applies the same concept as in [1] but allow the overcollection of losses to take place so that the efficient price signals are not dampened. As used in several regulatory and market mechanisms (e.g. Chile, Argentina, Uruguay, and NYISO in the United States) for transmission networks, we propose to use nodal factor pricing for distribution networks. The philosophy behind this idea is that as DG penetrates the distribution network we should consider it as an active network (i.e. like the transmission network) rather than as a passive network (i.e. a network which only has loads connected to it). The proposed method determines the prices at different nodes in the distribution network using nodal factors. These prices are short-run economically efficient and allocate losses based on location, time, and "net withdrawals". Moreover, these prices provide a much stronger economic signal for the location and installation of DG. Nodal factors are calculated using power flows locating "the reference bus" at the power supply point (PSP) where the transmission network connects to the distribution network. We assume no network constraints at the distribution level.

The paper is structured as follows. In Section 2 we will derive the nodal prices, nodal factors, and the merchandising surplus (MS) for the distribution network. In Section 3 we will consider a classical distribution network pricing approach focusing on how the costs associated with line losses are allocated among network users. In Section 4 we will present the new network distribution-pricing scheme that uses nodal factor pricing for an efficient losses cost allocation and compare the income that would accrue to the DG resource under the classical and the nodal factor pricing methods. In Section 5 we will present an application of the proposed method considering a rural radial distribution network. Finally, in Section 6 we will present some conclusions.

2 Nodal Factor Pricing in a Distribution Network

The manner in which we derive nodal factor prices in a distribution network is really no different from derving them for an entire power system. Define G_{k_g}, Q_{k_g} respectively, as the active and reactive power injected by generator k_g into busbar k_g, where the set of busbars k_g includes the power supply point (interface of transmission and distribution)and any busbars with distrbuted generation.

Define D_{k_e}, Q_{k_e} respectively, as the active and reactive power consumed by demand k_e and extracted from busbar k_e.

In order to simplify the notation, we assume that a busbar may only be a generating busbar or a demand busbar. In addition, we also assume that all power injections and extractions are independent of each other.

Let C_{k_g} be the total cost produced when $\left(G_{k_g}, Q_{k_g}\right)$ is injected into busbar k_g. In the same way, we may write,

$$C_{k_g} = C_{k_g}\left(G_{k_g}, Q_{k_g}\right)$$

where C_{k_g} is assumed to be convex, weakly increasing, and once continuously differentiable in both of its arguments.

The optimization problem for dispatching of distributed generation and supplying power at the power supply point can be represented as the following least-cost dispatch problem:

$$\min_{\substack{G_{k_g}, Q_{k_g}, D_{k_e}, Q_{k_e} \\ \forall k_g, k_e}} \sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g})$$

subject to the following constraints,

1) Electric balance:

$$Loss(G, D, Q) - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} = 0$$

2) Prime mover and thermal generators' constraints:

$$\begin{array}{c} 0 \leq G_{k_g} \leq \overline{G}_{k_g} \\ G_{k_g}^2 + Q_{k_g}^2 \leq \overline{S}_{k_g}^2 \end{array} \quad \forall k_g \leq n_g$$

Moreover, we will consider that $Loss(G, D, Q)$ is convex, increasing, and once continuously differentiable in all of its arguments. Under these hypothesis, application of the Karush-Kuhn-Tucker conditions lead to a system of equations and inequalities that guarantee the global maximum [6,10].

2.1 Nodal Prices

Assuming interior solutions and no network constraints, we obtain the following prices for active power from generations busbars, reactive power from generation busbars, active power at demand busbars, and reactive power at demand busbars respectively are:

$pa_{k_g} = \lambda(1 - \frac{\partial Loss}{\partial G_{k_g}})$

$pr_{k_g} = -\lambda(\frac{\partial Loss}{\partial Q_{k_g}})$

$pa_{k_e} = \lambda(1 + \frac{\partial Loss}{\partial D_{k_e}})$

$pr_{k_e} = \lambda(\frac{\partial Loss}{\partial Q_{k_e}})$

λ is the active power price at the PSP

These prices define the economic dispatch [11] and correspond to what it is widely known as nodal pricing. However, only prices for active energy are generally used in actual regulated or competitive markets, disregarding those for reactive energy.

2.2 Nodal Factors (NFs)

As seen before, the active energy marginal prices result (without regarding the constraints) from the product of λ by the factor,

$(1-\frac{\partial Loss}{\partial G_{k_g}})$, in the case of a generator busbar, and

$(1+\frac{\partial Loss}{\partial D_{k_e}})$, in the case of a demand busbar.

If we make the following change of variables, $P_k = D_{k_e}$ and $P_k = -G_{k_g}$, it results, $pa_k = \lambda(1+\frac{\partial Loss}{\partial P_k})$. Therefore, we define $fn_k = 1+\frac{\partial Loss}{\partial P_k}$ as the Active Nodal Factor (also called Penalty Factor [11]) corresponding to busbar *k* ($pa_k = \lambda fn_k$).

In the same way, it is possible to define the Reactive Nodal Factor for busbar *k* as $fn'_k = \frac{\partial Loss}{\partial Q_k}$ ($pr_k = \lambda fn'_k; Q_k = Q_{k_e} = -Q_{k_g}$).

We observe that the partial derivative of the power system losses with respect to the extracted active and reactive power at busbar *k* must be evaluated at the values of the electrical variables that correspond to the steady state equilibrium point for a given optimal dispatch.

2.3 Merchandising Surplus (MS)

Using the same notation as before, we can define the merchandising surplus as:

$$MS = \sum_{k_e=1}^{n_e} pa_{k_e} D_{k_e} - \sum_{k_g=1}^{n_g} pa_{k_g} G_{k_g} + \\ + \sum_{k_e=1}^{n_e} pr_{k_e} Q_{k_e} - \sum_{k_g=1}^{n_g} pr_{k_g} Q_{k_g}$$

The *MS* results from the difference between the amount of money paid by consumers and the amount of money received by generators. It is calculated using the same time basis as prices (i.e. if prices are set hourly, then *MS* is calculated hourly).

It is possible to prove that for a network without constraints, the *MS* is approximately equal to the cost of losses:

If no constraints are operating in the network, then, $pa_{k_g} = \lambda(1-\frac{\partial Loss}{\partial G_{k_g}}), pr_{k_g} = -\lambda(\frac{\partial Loss}{\partial Q_{k_g}}), pa_{k_e} = \lambda(1+\frac{\partial Loss}{\partial D_{k_e}}), pr_{k_e} = \lambda(\frac{\partial Loss}{\partial Q_{k_e}})$

Then,

$$MS = \sum_{k_e=1}^{n_e} \lambda(1+\frac{\partial Loss}{\partial D_{k_e}}) D_{k_e} - \sum_{k_g=1}^{n_g} \lambda(1-\frac{\partial Loss}{\partial G_{k_g}}) G_{k_g} +$$

$$+ \sum_{k_e=1}^{n_e} \lambda(\frac{\partial Loss}{\partial Q_{k_e}}) Q_{k_e} + \sum_{k_g=1}^{n_g} \lambda(\frac{\partial Loss}{\partial Q_{k_g}}) Q_{k_g}$$

or,

$$MS = \lambda[\sum_{k_e=1}^{n_e} D_{k_e} - \sum_{k_g=1}^{n_g} G_{k_g}] +$$

$$+\lambda[\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial D_{k_e}} D_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial G_{k_g}} G_{k_g}] +$$

$$+\lambda[\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} Q_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} Q_{k_g}]$$

Noting that the first term equals $-\lambda Loss$ and the summation of the last four terms is a linear approximation of two times losses (that could be greater or less than actual losses), multiplied by λ , it results that:

$$MS \simeq -\lambda Loss + 2\lambda Loss$$

$$MS \simeq \lambda Loss$$

Of course, if there were binding network constraints, then *MS* could be much more the cost of losses.

3 Classical Network Distribution Pricing Scheme: Averaging Losses

Let λ be the wholesale electricity price at the power supply point busbar (interface between transmission and distribution). The Disco revenue for that network, which is established by the regulator, is composed by the capital costs and the operational costs. Generally, within a classical network pricing approach, both types of costs are summed up and averaged among all customers on a per kWh basis. This can be summarized as follows:

$$R = R_{cap} + R_{op}$$

where, R is the Disco regulated revenue, R_{cap} is the revenue related with capital costs, and R_{op} is the revenue related with operational costs. It is worth observing that,

$$R_{op} = R_{loss} + R_{Noloss}$$

where, R_{loss} is the revenue related with loss costs and R_{Noloss} is the revenue related with operational costs different from losses.

A simple tariff formula, for a classical network pricing approach would be:

$$T = \frac{R}{\sum_j E_j},$$

where E_j is the active energy demand (or generation) of customer j in the measurement time period and T is the distribution use of system (DUS) tariff on a per kWh basis.

As $R = R_{cap} + R_{loss} + R_{Noloss}$, then it is possible to decompose tariff T in a similar manner:

$$T = T_{cap} + T_{loss} + T_{Noloss}$$

where, $T_{cap} = \frac{R_{cap}}{\sum_j E_j}$, $T_{loss} = \frac{R_{loss}}{\sum_j E_j}$, and $T_{Noloss} = \frac{R_{Noloss}}{\sum_j E_j}$.

Within this scheme a demand type customer *i* connected to the network would pay λ (USD/kWh) for the active energy, plus the transmission use of system (TUS) charges (e.g. in USD/kWh), plus the DUS tariff (USD/kWh).

On the other hand, a DG *k* connected to the network would pay TUS charges, plus T , getting λ for the active energy sold.

As a result, the method used for allocating the cost of losses in this case is just averaging them among all customers (generators or loads) throughout the tariff: $T_{loss} = \frac{R_{loss}}{\sum_j E_j}$.

Consequently, this network-pricing scheme gives no consideration for individual customers such as DG, which may reduce the amount of losses. This fact is not a surprise since this type of formula is designed for customers that only consume electricity (i.e. within a passive network philosophy).

Moreover, in some regulatory environments that explicitly recognise DG, such as in Uruguay, DG is exempted from paying network charges. Therefore R_{loss} is averaged only among consuming customers. While this provides an incentive for DG to be deployed, it still does not send the right price signals regarding the location of the DG resource.

4 A New Network Distribution Pricing Scheme: Allocating Losses with Nodal Prices

The idea of the proposed method is to recover the cost of losses using the merchandising surplus through the nodal energy prices that also serve as the allocation mechanism. The nodal pricing mechanism uses the whole *MS* to pay part of the Disco revenue (*R*), approximately covering losses, and to allocate the remaining revenue with a classical tariff formula.

If nodal prices of Section II are used in the distribution network, then there is a merchandising surplus, *MS* that is approximately equal to the cost of losses (as seen in Section II). In general (but not necessarily), *MS* is greater than the cost of losses and thus it is possible to recover a bit more than R_{loss} through nodal prices. Consequently, the remaining revenue to be collected for this case is:

$$R_{rem} = R - MS$$

R_{rem} can be allocated among customers using, for example, the simple classic tariff formula:

$$T_{rem} = \frac{R - MS}{\sum_j E_j},$$

thereby allowing the distribution utility to collect its required revenue. The difference will be seen with respect to the energy prices charged, which will now include marginal losses, to consumers and DG resources that were derived in Section II.

4.1 Classical vs. Proposed Network Distribution Pricing

Within the nodal pricing scheme a DG *k* connected to the network would pay TUS charges, plus T_{rem}, getting $pa_k = \lambda f n_k$ for the active energy sold and $pr_k = \lambda f n'_k$ for the reactive energy sold. For the sake of simplicity we will suppose that the generator produces at constant active power G_k and reactive power Q_k for H_k hours. As a result, the net income for DG *k* would be:

$$NI_{A1_k} = [\lambda f_{n_k} G_k + \lambda f'_{n_k} Q_k - TUSG_k - T_{rem} G_k] \times H_k$$

On the other hand, within the classical scheme the net income would be:

$$NI_{C_k} = [\lambda G_k - TUSG_k - TG_k] \times H_k$$

Consequently, the difference between alternative 1 and the classical scheme would be,

$$\Delta NI_{A1C_k} = NI_{A1_k} - NI_{C_k}$$

$$= [\lambda(f_{n_k} - 1)G_k + \lambda f'_{n_k} Q_k + (T - T_{rem})G_k] \times H_k$$
$$= [\lambda(\frac{\partial Loss}{\partial P_k})G_k + \lambda \frac{\partial Loss}{\partial Q_k} Q_k + (\frac{MS}{\sum_j E_j})G_k] \times H_k$$

As it can be seen ΔNI_{A1C_k} becomes greater as DG reduces network losses, thus giving the appropriate signals. In the case of a distribution network with no (or small) penetration of DG (where all the power flux is from the PSP to the loads), it can be observed that if DG *k* is operating at lagging power factor (i.e. delivering both active and reactive power to the network) then, ΔNI_{A1C_k} is always greater than zero as each of the summation terms is greater than zero. ΔNI_{A1C_k} is composed by the value of the contribution of DG *k* to loss reduction ($\lambda(\frac{\partial Loss}{\partial P_k})G_k H_k$)+($\lambda \frac{\partial Loss}{\partial Q_k} Q_k H_k$), plus a fraction of the *MS* produced in the distribution network by the application of nodal prices.

5 An example

Let us consider the rural radial distribution network of Fig. 1 with details shown in Tables 1 and 2. The characteristics of the distribution network are meant to reflect conditions in Uruguay where there are potentially long, radial lines. This network consists of a busbar (1) which is fed by a 150/30 kV transformer, and 4 radial feeders (A, B, C, D). For the purpose of simplicity, we will just consider feeder A for our calculations. Feeder A consists of a 30 kV overhead line feeding 6 busbars (3, 4, 5, 6, 7, 8). Except for the case of busbar 4, which is an industrial customer, all the other busbars are 30/15 kV substations providing electricity to low voltage customers (basically residential).

Table 1. Typical data for 120AlAl conductor

$r(\Omega/km)$	$x(\Omega/km)$
0.3016	0.3831

Table 2. Information data for the rural radial distribution network

Send. bus	Rec. bus	Length (km)	Type of Cond.
1	2	10.0	120AlAl
2	3	1.6	120AlAl
2	4	26.0	120AlAl
4	5	3.0	120AlAl
5	6	1.5	120AlAl
6	7	5.6	120AlAl
7	8	13.5	120AlAl

The daily load profiles for the busbars are shown in Fig. 2 are also reflective of what might be observed in Uruguay. We will assume then that residential customers have the simplified load profile of Fig. 2.A and the industrial customer the simplified load profile of Fig. 2.B.

There are four different scenarios depending on the time of the day:

i. SI, from 0 to 7, summing up 7 hours;
ii. SII, from 7 to 18, summing up 11 hours;
iii. SIII, from 18 to 22, summing up 4 hours;
iv. SIV, from 22-24, summing up 2 hours.

It would have been also possible to include seasons in the load modelling, but for simplicity we have just consider only one. We will assume that prices at busbar 1 for the 4 scenarios are: $\lambda_{SI} = 16USD/MWh$, $\lambda_{SII} = \lambda_{SIV} = 24USD/MWh$, $\lambda_{SIII} = 30USD/MWh$ which are reflective of power prices in Uruguay during these time periods.

Computation [7] of the network in this case leads to the results of Tables 3, 4, 5 and 6).

As it can be seen, for this case the *MS* sums up (SI, SII, SIII, and SIV together) 98,423 USD/year while total cost for losses are 75,243 USD/year. As expected, the *MS* recovers more of the loss cost.

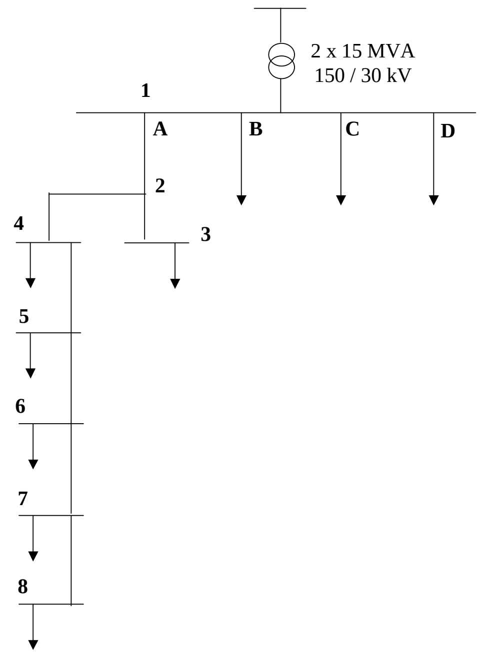

Figure 1. A rural distribution network.

Let us consider now the same distribution network of Fig. 1, but with a distributed generator connected to busbar 8.

The DG resource at busbar 8 is a 1 MVA synchronous generator operating at 0.95 lagging power factor. We assume this distributed generation unit runs in all hours and has a cost that is below the system price at all hours. Computation [7] of the network in this case leads to the results in the far right columns of of Tables 3, 4, 5 and 6.

In this case, *MS* sums up (SI, SII, SIII and SIV together) 57,560 USD/year, while total cost for losses are 46,986 USD/year. Once again, the *MS* recovers more than the cost of losses. It can be seen that in this case (with DG), the *MS* is closer the value of the loss cost than was the case without the DG resource. This is because the DG resource at busbar 8 reduces the network losses and consequently the approximation of the merchandising surplus converges toward the cost of losses. In addition, it is interesting to note that for SI, when the distribution network is exporting power to the grid, *MS* recovers less than the losses cost.

Let us consider the DG's income within the nodal pricing scheme:

$$I_{NP}(G) = 210448USD/year$$

Otherwise, within a classical scheme, the DG re-

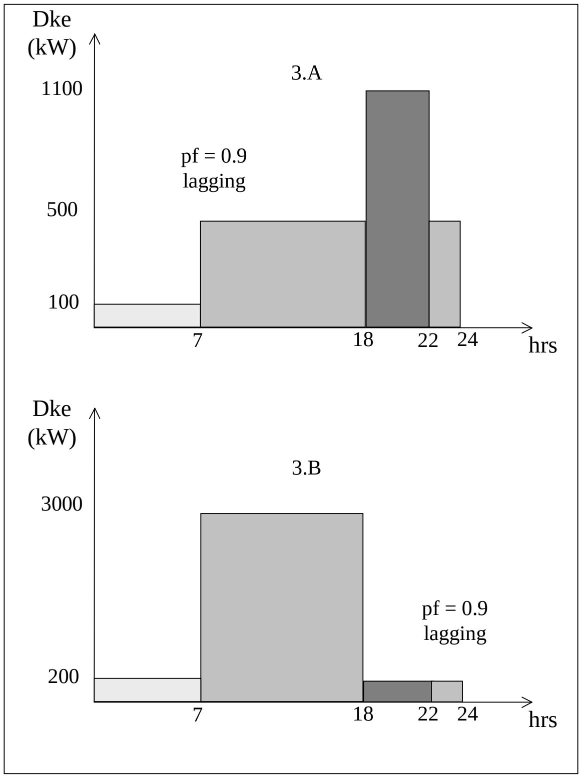

Figure 2. Daily load profiles.

source would get:

$$I_C(G) = 188632 USD/year$$

As a result, the nodal pricing scheme provides G, around 10 % more income than the classical scheme, where energy prices are identical in all busbars.

If we evaluate the difference of net income as defined in Section 4.1, it results:

$$\Delta NI_{A1C} = 33091 USD/year$$

Finally, it is interesting to observe the implications that the connection of the DG resource produces in the network:

i. Losses drop from 2946 MWh/year to 1845 MWh/year (37 % less)

ii. Maximum voltage drop decreases from 13.9 % to 10.4 %

iii. Maximum current through the overhead line is reduced from 137 A to 112 A (thus reducing the line utilisation by 18 %).

6 Conclusion

This paper has presented the widely used nodal pricing scheme applied to distribution networks. We have shown that this economically efficient scheme provides better incentives for the deployment of DG than simple loss averaging. The increased incentiove arises since DG is paid for the reduction of losses and for the provision of reactive service. DG transforms the distribution network into an "active network" like the transmission system. As a result, the treatment of both types of networks should become the same at some point in the future. Further work will assess in detail a practical implementation of the nodal factor pricing method.

Table 3. Results for SI

	No DG		DG	
Bus	p_a	p_r	p_a	p_r
1	16	0	16	0
3	16.0784	0.0384	15.976	0.0048
4	16.2512	0.1232	15.8816	0.0032
5	16.264	0.1296	15.864	0
6	16.2704	0.1328	15.8544	-0.0032
7	16.2832	0.1392	15.8096	-0.016
8	16.2976	0.1456	15.6928	-0.0512

	No DG	DG
$MS(USD/yr)$	270.5	252.6
$\Delta V(\%)$	1.47	1.2
$Losses(MWh/yr)$	16.6	16.3
$Loss(USD/yr)$	265.7	260.8
I_{max}	15.1	16.9

Table 4. Results for SII

	No DG		DG	
Bus	p_a	p_r	p_a	p_r
1	24	0	24	0
3	25.14	0.6864	24.8952	0.54
4	28.2648	2.3688	27.2136	1.8408
5	28.428	2.4504	27.2904	1.8888
6	28.488	2.4816	27.3096	1.9032
7	28.644	2.556	27.3168	1.9272
8	28.8336	2.6496	27.1704	1.9056

	No DG	DG
$MS(USD/yr)$	65091.6	39115.5
$\Delta V(\%)$	12.8	9.7
$Losses(MWh/yr)$	2075.8	1327.0
$Loss(USD/yr)$	49818.1	31846.8
I_{max}	132.2	108.8

Table 5. Results for SIII

	No DG		DG	
Bus	p_a	p_r	p_a	p_r
1	30	0	30	0
3	31.503	0.9	31.182	0.702
4	35.118	2.901	33.771	2.184
5	35.571	3.129	34.083	2.349
6	35.742	3.216	34.191	2.409
7	36.183	3.432	34.41	2.541
8	36.732	3.702	34.473	2.634

	No DG	DG
$MS(USD/yr)$	31042.7	17495.0
$\Delta V(\%)$	13.9	10.4
$Losses(MWh/yr)$	778.5	474.1
$Loss(USD/yr)$	23354.2	14223.7
I_{max}	137.0	112.0

Table 6. Results for SIV

	No DG		DG	
Bus	p_a	p_r	p_a	p_r
1	24	0	24	0
3	24.492	0.2616	24.312	0.1824
4	25.5696	0.8136	24.8784	0.5328
5	25.6896	0.8736	24.9360	0.5688
6	25.7352	0.8952	24.9504	0.5808
7	25.848	0.9504	24.9552	0.6000
8	25.9872	1.0176	24.8448	0.5832

	No DG	DG
$MS(USD/yr)$	2017.8	696.8
$\Delta V(\%)$	6.0	3.3
$Losses(MWh/yr)$	75.2	27.3
$Loss(USD/yr)$	1804.7	654.2
I_{max}	60.6	39.8

References

[1] Mutale, J., Strbac, G., Curic, S. and Jenkins, N. Allocation of losses in distributions systems with embedded generation, *IEE Proc.-Gener. Transm. Distrib.* , Vol. 147, N 1, 2000, pp. 1-8.

[2] Kirschen, D., Allan, R. And Strbac, G. Contribution of Individual Generators to Loads and Flows, *IEEE Transactions on Power Systems*, Vol. 12, N 1, 1997.

[3] Bialek, J. Topological Generation and Load Dispatch Factors for Supplement Charge Allocation in Transmission Open Access, *IEE Transactions on Power Systems*, Vol. 12, N 3, 1997, pp. 1185-1194.

[4] Wu, F. F. and Varaiya, P. Co-ordinated Multilateral Trades for Electric Power Networks: Theory and implementation, *University of California Energy Institute*, Power Report PWP-031, 1995.

[5] Gross, G. and Tao, S. A loss Allocation Mechanism for Power System Transactions, presented at *Bulk Power System Dynamics and Control IV - Restructuring*, Santorini, Greece.

[6] Vignolo, M. and Zeballos, R. Economic Operation of Power Systems. *Proceedings Latin America IEEE T & D 2002*, N 19, 127, 2002.

[7] Ghosh, S. And Das, D. Method for load-flow solution of radial distribution networks, *IEE Proc.-Gener.Transm.Distrib.*,Vol. 146, N 6, November 1999, pp. 641-648.

[8] Thomas Ackermann. ”Distributed Generation - A Definition”, *Proceedings of the International Symposium on Distributed Generation: Power System and Market Aspects*, Stockholm, Sweden, June 11-13, 2001.

[9] CIGRE (1999). Impact of Increasing Contribution of Dispersed Generation on Power System, Working Group 37.23.

[10] G.L. Nemhauser, A.H.G. Rinnoy Kay, M.J. Todd (eds.), *Handbooks in Operation Research and Management Science*, Vol. 1, Optimization, North Holland, 1989.

[11] Momoh, J. *Electric Power System Applications of Optimisation*. ISBN: 0-8247-9105-3. Marcel Deker, 2001.