Allocation of Fixed Costs in Distribution Networks With Distributed Generation

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***Abstract*—In this paper, we propose a method for the allocation of fixed (capital and nonvariable operation and maintenance) costs at the medium voltage (MV) distribution level. The method is derived from the philosophy behind the widely used MW-mile methodology for transmission networks that bases fixed cost allocations on the "extent of use" that is derived from load flows. We calculate the "extent of use" by multiplying the total consumption or generation at a busbar by the marginal current variations, or power to current distribution factors (PIDFs) that an increment of active and reactive power consumed, or generated in the case of distributed generation, at each busbar, produces in each circuit. These PIDFs are analogous to power transfer distribution factors (PTDFs).**

Unlike traditional tariff designs that average fixed costs on a per kWh basis across all customers, the proposed method provides more cost-reflective price signals and helps eliminate possible cross-subsidies that deter profitable (in the case of competition) or cost-effective (in the case of a fully regulated industry) deployment of DG by directly accounting for use and location in the allocation of fixed costs. An application of this method for a rural radial distribution network is presented.

***Index Terms*—Allocation of fixed costs, distributed generation (DG), distribution networks.**

I. Introduction

It is becoming widely accepted that distributed generation (DG) resources can provide benefits to distribution and transmission networks, reducing line losses, acting as a network service provider by postponing new distribution or transmission reinforcements, and providing ancillary services. In addition, as a modular technology, it may present a lower cost addition to the system in that a large facility need not to be built that has excess capacity for some years. The Working Group 37.23 of CIGRE has summarized in [1] some of the reasons for an increasing share of DG in different countries.

As it is likely that DG will become more prevalent in distribution systems, we are interested in modeling the distribution network with DG paying particular attention to the design of tariffs for the recovery and allocation of distribution network fixed costs, including capital and nonvariable operation and maintenance (O & M) costs. It is already well understood that nodal energy prices as developed by [2] send short-run efficient time and location differentiated price signals to load and generation in transmission networks as discussed in [3]. These signals can also be used for sending the appropriate signals for the siting of DG in distribution networks as demonstrated in [4]. While these short-run efficient nodal prices collect more revenue from loads than is paid out to generators, it has been shown in [5]–[7] to be insufficient to cover the remaining infrastructure and other fixed costs of the network.

It is also well established that passing through the remaining infrastructure costs on a pro rata basis, as is often the case in many tariff methodologies, does not provide price signals that are based on cost causality (cost reflective) or are long-run efficient for investment in new network infrastructure, or for the location of new loads or generation. Beginning with [8], many have written about "extent of use" methods for the allocation of transmission network fixed costs. These "extent of use" methods for allocating costs have also become known generically as MW-mile methods, as they were called in [8]. The "extent of use" can be generically defined as a load's or generator's impact on a transmission asset (line, transformer, etc.) relative to total flows or total capacity on the asset as determined by a load flow model. Other variations on this same idea can be seen in [9]. An interesting trend in the literature on MW-mile methodologies emerges on closer examination. As different methods are proposed to allocate fixed transmission costs, rarely is there any incentive to provide for counter-flow on a transmission asset, the contention being that transmission owners would be against making payments to generators that provided counter-flows and the worry that the method would no longer be revenue sufficient [8], [10]–[12]. Reference [10] allows for counterflows but, to ease potential worries to transmission owners, proposes that counterflows be assessed a charge of zero.

As there are many cost allocation methods, there are many load flow-based methods to determine the extent of use. References [13]–[15] use a tracing method that relies on the use of proportional sharing of flows into and out of any node. Marginal factors such as distribution factors are used in [8] and [6], while [16] uses a line utilization factors that depend on demand in the system being held constant. Reference [12] provides an overview and comparison of these methods and shows that they arrive at very similar results for flows and charges, leading them to conclude that there still is no agreement on the best method to determine the extent of use.

As discussed in [4], the rationale behind this paper is that the presence of DG in the distribution network transforms distribution from a passive network (e.g., a network that only has loads connected to it) into an active network, not unlike a transmission network. As with nodal pricing for short-run operation of power

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systems where price signals are sent so that generators close to loads are rewarded for reducing losses, or generators locating downstream of a congested asset are rewarded for alleviating that congestion, *generators or loads that locate in a manner that reduces line loading or uses fewer assets should be rewarded with lower charges for the recovery of fixed costs as essentially these generators or loads "create" additional distribution capacity*. As a result, we propose that extent of use cost allocation methodologies from transmission networks could, and should, be adopted to promote more cost-reflective pricing, which will provide better financial incentives for the entry and location of DG or large loads on, and investment in, distribution networks.

Our extent of use measure uses marginal changes in current, as opposed to power, in a distribution asset with respect to both active and reactive power injections multiplied by those injections to determine the extent of use at any time t.[1] Unlike most previous applications of extent of use measures, our extent of use measure explicitly accounts for flow direction to provide better long-term price signals and incentives for DG to locate optimally in the distribution network and to alleviate potential constraints and reduce losses.

We propose two possibilities to price the extent of use, the merits of which will be discussed in the next section. First, we can compute the extent of use at each bus in each hour and will price the extent of use on a per MWh basis at each bus in each hour, with any remaining fixed costs spread over all load in the system on a per MWh basis. The other pricing option we explore is the use of fixed charges based on the extent of use at each bus at the system coincident peak, with any remaining fixed costs recovered over all load at coincident peak.

The paper is structured as follows. In Section II, we will present the general allocation strategy. In Section III, we will present the proposed electricity "extent of use" measurement methodology. In Section IV, we will present an application of the proposed method considering a rural radial medium-voltage (MV) distribution network with results presented in Section V. Section VI concludes.

II. Allocation Strategy and Description of Charges

From an economic perspective, allocation methods for fixed costs do not have efficiency properties *per se*. However, the allocation of costs, regardless of the method, is entirely necessary for the owners of distribution infrastructure so they may recover the costs associated with providing distribution service. *Thus, given the general lack of efficiency properties and the need to allocate fixed costs, allocating costs to those who cause them (cost causality) is another method that is often used and is the criteria we use in our allocation strategy.* Moreover, since these are fixed costs that are being allocated, there are no "short-term" incentive changes that one would observe akin to the changes that occur when moving to efficient nodal prices for energy.

However, there are long-term entry and siting incentives that may change, depending on the fixed cost allocation. Consider the siting of DG on a distribution system. An allocation of costs based on the line loading attributable to DG that pays generators for providing counter flow that effectively "creates" additional capacity provides a better financial incentive to DG to site where it provides counter flow versus siting at a location where it increases line loading, all else equal, or for a large industrial customer, allocation of costs based on the extent of use will lead it to site its facility closer to the interface with the transmission system rather than at the end of the network, where it will increase loading for more facilities.

In the design of our allocation strategy, we make two observations regarding distribution networks. The first is that distribution networks are designed primarily to handle circuit currents. The second observation is that current flow better corresponds to the thermal capacity limits of a line or asset since voltages may not necessarily be held constant in the network [17]. Consequently, the "extent of use" of distribution network circuits can be measured in terms of the contribution of each customer to the current flow, not the power flow, through the circuit at any point in time similar to [18] in their derivation of utilization factors. This current flow can be traced to injections and withdrawals of active and reactive power at each busbar using active and reactive power to current distribution factors, APIDFs and RPIDFs, respectively. Our extent of use measure is grounded in the idea that costs should be allocated to those who cause them. Given that we propose to use current flows attributed to network customers, we choose to call our methodology an "Amp-mile" or "I-mile" methodology for allocating fixed distribution network costs.

The contribution of a given customer to the current flow on a given circuit at any time is the summation of the correspondent APIDF and RPIDF times the actual active and reactive power, respectively, injected or withdrawn by the customer at that time. The summation for a given circuit of all customers contributions closely approximates the current flow. A reconciliation factor must be used to obtain the exact current flow through the circuit using the APIDFs and the RPIDFs. The reconciliated contributions can be used as a measure of the "extent of use," and active power extent of use (AEoU) and reactive power extent of use (REoU) factors can be obtained.

The fixed cost of each circuit is calculated summing up the capital and nonvariable O & M costs of the conductor and other circuit related equipment such as circuit breakers, isolators, dischargers, etc., including installation costs. The capital portion of the fixed cost is assumed to be a levelized cost. A locational charge for each customer, which recovers the used network capacity, can be determined summing up the individual facility charges for circuit usage. These individual charges are obtained multiplying the correspondent AEoU and REoU factors by the adapted circuit cost (ACC). The ACC for a circuit is calculated multiplying the levelized circuit cost by the used circuit capacity (UCC) factor, which is given by the ratio between current flow and current capacity of the circuit. As suggested by [9] and [12], and employed by [14], any remaining network costs related to the unused capacity of the circuits can be recovered by a nonlocational charge.

We obtain for each customer (generator/demand) two types of charges, locational and nonlocational, for the recovery of fixed costs for the distribution network. The first is a locational

[1]The extent of use measure we propose is not a marginal methodology like the nodal pricing of congestion and losses but is analogous to the expenditures incurred or revenues gained (price multiplied by quantity) under nodal pricing.

charge, based on the extent of use, that should be paid to cover the portion of fixed cost for network service considering both active power (active locational charge) and reactive power (reactive locational charge) injections or withdrawals. Unlike previous applications of flow-based extent of use methodologies and charges that only account for flow magnitudes and not flow direction, *in our Amp-mile method, we explicitly account for counterflows and reward potential DG units that free up or, in effect, create additional distribution network capacity with negative locational payments (payments to the DG source).* The second charge is a nonlocational charge that is levied to recover the cost of the unused network capacity and spreads the cost of the unused capacity over all load in some fashion.[2] It can be argued that the spare capacity can be seen as a common "system benefit" to all users as the excess capacity reduces losses for every customer and provides system security and therefore should be paid for by all users.

There exist a variety of possibilities for assessing the locational and nonlocational charges. One possibility is to allocate both charges on a per MWh basis. However, a drawback to allocating charges for fixed costs on a per MWh basis is that it would distort short-term price signals if those short-term signals were based on efficient nodal prices. However, assessing the charges on a per MWh basis would make it easier to implement the suggestion by [19] that extent of use charges for network infrastructure may be more long-term efficient if they are time differentiated to account for different usages patterns over different time periods. By assessing these charges in each hour, we are taking the suggestion to the extreme. Time differentiating locational charges for the recovery of fixed costs has also been previously implemented in [20]. At the other extreme, the charges could be assessed as a fixed charge. The basis for the fixed locational charge could be determined by a customer's contribution to line loading at system peak, while the remaining nonlocational charge could be based on the demand at coincident peak. The main rationales for a fixed charge are that it holds with the logic of distribution network design to serve the system peak and fixed charges also preserve the efficiency of short-term nodal prices. There are other possibilities for allocating fixed charges, but that is beyond the scope of this paper.

In our application in Sections IV and V, we will provide examples using both per MWh charges and fixed charges based on demand at system peak for both the locational component and the nonlocational component.

III. Extent of Use Measurement Methodology and Charges

A. *Defining the Extent of Use*

In [17], the power to current distribution factor, from injection at bus k to current magnitude on the line l, is defined as the sensitivity

$$\frac{\partial \overline{I}_l}{\partial P_k}. \tag{1}$$

[2]We allocate nonlocational charges over only load as this is the tariff method used in Uruguay, where our example is based in Section IV. If we allocated some of these costs to generators, it would not change our results qualitatively.

We define active power to absolute current distribution factor with respect to an injection or withdrawal at bus k to the absolute value of current on the line l, at time t, as the sensitivity

$$APIDF_{lk}^t = \frac{\partial I_l^t}{\partial P_k^t} \tag{2}$$

where I_l^t is the absolute value of current $\overline{I}_l^t$ through circuit l, at time t, and P_k^t is the active power withdrawal at node k, at time t.

In the same way, the reactive power to absolute current distribution factor with respect to an injection or withdrawal at bus k to absolute value of current on the line l, at time t, can be defined as the sensitivity

$$RPIDF_{lk}^t = \frac{\partial I_l^t}{\partial Q_k^t} \tag{3}$$

where Q_k^t is the reactive power withdrawal at node k, at time t.

Within this framework, both $APIDF_{lk}^t$ and $RPIDF_{lk}^t$ are calculated using the Jacobian matrix derived from the power flow equations of Appendix B.

Absolute value of current at line l, at time t, can be approximated as

$$I_l^t \cong \sum_{k=2}^{n} APIDF_{lk}^t \left[PL_k^t + PG_k^t\right] + \sum_{k=2}^{n} RPIDF_{lk}^t \left[QL_k^t + QG_k^t\right] \tag{4}$$

where

- PL_k^t active power consumption of a demand customer at busbar k, for time t with $PL_k^t \geq 0$;
- PG_k^t active power consumption of a generation customer at busbar k, for time t with $PG_k^t < 0$;
- QL_k^t reactive power consumption of a demand customer at busbar k, for time t with $QL_k^t \geq 0$;
- QG_k^t reactive power consumption of a generation customer at busbar k, for time t with $QG_k^t < 0$ for a generator providing reactive power to the network;
- n number of busbars in the distribution network, with $k = 1$ as the slack bus and m is the number of lines in the network, where $m \leq n - 1$.

I_l^t turns out to be a close approximation as circuit currents are approximately a linear function of active and reactive power at busbars. However, to define AEoU and REoU factors, a reconciliation factor is needed so that the "extent of use" factors for a given line sum to 1. We define AI_l^t so that

$$AI_l^t = \sum_{k=2}^{n} APIDF_{lk}^t \left[PL_k^t + PG_k^t\right] + \sum_{k=2}^{n} RPIDF_{lk}^t \left[QL_k^t + QG_k^t\right]. \tag{5}$$

Then, dividing by AI_l^t, the product of the active/reactive power to current distribution factor with the active/reactive power injection or withdrawal, we obtain extent of use factors.

Note that the summation for all busbars, for a given line l, at a given time t, of these factors equals one.

Active power-related extent of use factor for line l with respect to demand at busbar k, for time t:

$$AEoUL_{lk}^t = \frac{APIDF_{lk}^t \times PL_k^t}{AI_l^t}. \quad (6)$$

Active power-related extent of use factor for line l with respect to generation at busbar k, for time t:

$$AEoUG_{lk}^t = \frac{APIDF_{lk}^t \times PG_k^t}{AI_l^t}. \quad (7)$$

Reactive power-related extent of use factor for line l with respect to demand at busbar k, for time t:

$$REoUL_{lk}^t = \frac{RPIDF_{lk}^t \times QL_k^t}{AI_l^t}. \quad (8)$$

Reactive power-related extent of use factor for line l with respect to generation at busbar k, for time t:

$$REoUG_{lk}^t = \frac{RPIDF_{lk}^t \times QG_k^t}{AI_l^t}. \quad (9)$$

B. *Defining the Costs and Charges*

Let CC_l be the levelized annual cost of circuit l. If line flows are measured every hour during the year, for example, then the levelized cost for each hour $CC_l^t = CC_l/8760$. Without loss of generality, the number of time periods can vary depending on how often flows are measured, whether they be every hour or every 5 min.

The adapted cost of circuit l, for time t, is defined as

$$ACC_l^t = UCC_l^t \times CC_l^t \quad (10)$$

where UCC_l^t is the used circuit capacity of l, for time t, and is defined by

$$UCC_l^t = \frac{I_l^t}{CAP_l} \quad (11)$$

where I_l^t is the current through circuit l, for time t, and CAP_l, the circuit capacity of l.

1) Time Differentiated Per Unit Charges: Related active and reactive locational charges for demand/generation at busbar k, for time t, can now be determined. These charges can be expressed as a total charge at time t, though given that these charges could change on an hourly basis, they are for all intents and purposes time differentiated per MWh or MVArh charges and is the way we shall express the charges below.

The total **active locational charge for demand at bus k:**

$$AL_k^t = \sum_{l=1}^{m} AEoUL_{lk}^t \times ACC_l^t. \quad (12)$$

The total charge can be broken down into a per MWh charge by noting that total charges for bus k can be expressed as

$$AL_k^t = \sum_{l=1}^{m} \frac{APIDF_{lk}^t \times PL_k^t}{AI_l^t} \times \frac{I_l^t}{CAP_l} CC_l^t. \quad (13)$$

Note that $AI_l^t \cong I_l^t$ for each line l, and dividing through by the active power demand at bus k, PL_k^t, then the per MWh charge can be expressed as

$$\frac{AL_k^t}{\text{MWh}} \cong \sum_{l=1}^{m} \frac{APIDF_{lk}^t \times CC_l^t}{CAP_l}. \quad (14)$$

As a time and location differentiated charge, the per unit charge has two desirable properties in terms of cost causality. First, as active power load at bus k increases, the extent of use increases so that at peak usage times, the customer at bus k will face a higher overall charge. Second, the more circuits over which power demanded at bus k must travel, the greater will be the overall charge.

Moreover, the per unit charges, a per MWh charge as expressed in (14), should be stable over both time and differing load levels at bus k. Both CC_l^t and CAP_l are constants. Also, $APIDF_{lk}^t$ is approximately constant as the relationship between injections or withdrawals and current flow are approximately linear.

Analogously, for active power injected, **the total active locational charge for generation at bus k:**

$$AG_k^t = \sum_{l=1}^{m} AEoUG_{lk}^t \times ACC_l^t \quad (15)$$

and just as we have define the per MWh charge for load, the per MWh charge for generation at bus k is

$$\frac{AG_k^t}{\text{MWh}} \cong -\sum_{l=1}^{m} \frac{APIDF_{lk}^t \times CC_l^t}{CAP_l}. \quad (16)$$

Note that for this case, a minus sign must be added in the formula because PIDFs are defined for the case of withdrawals and power generation, PG_k^t, is a negative withdrawal when calculating this per MWh charge. Then, when the generation at bus k is providing counterflow, the per MWh charge for injections at bus k are really payments made to generation for "creating" extra capacity on each circuit l. The more circuits for which counterflows are created and hence "capacity created" also implies that this payment increases.

We can now define analogous charges for reactive power withdrawals and injections at bus k that have the same properties and interpretations.

Related reactive locational charge for demand at bus k:

$$RL_k^t = \sum_{l=1}^{m} REoUL_{lk}^t \times ACC_l^t \quad (17)$$

$$\frac{RL_k^t}{\text{MWh}} \cong \sum_{l=1}^{m} \frac{RPIDF_{lk}^t \times CC_l^t}{CAP_l}. \quad (18)$$

Related reactive locational charge for generation at bus k:

$$RG_k^t = \sum_{l=1}^{m} REoUG_{lk}^t \times ACC_l^t \tag{19}$$

$$\frac{RG_k^t}{\text{MWh}} \cong -\sum_{l=1}^{m} \frac{RPIDF_{lk}^t \times CC_l^t}{CAP_l}. \tag{20}$$

2) Fixed Charges Based on Extent of Use at System Peak: Fixed charges based on the extent of use at the system peak have two desirable attributes over per unit charges. First, as the charge is independent of use at each hour except the peak hour, it will not distort efficient short-term price signals such as nodal prices. Second, as distribution networks are often designed explicitly to handle the system peak, it is logical to assess the charge based on use at the peak. Consider our measure of the extent of use defined in (6)–(9) and define the extent of use at system peak for active and reactive load and generation as

$$AEoUL_{lk}^{peak} = \frac{APIDF_{lk}^{peak} \times PL_k^{peak}}{AI_l^{peak}} \tag{21}$$

$$AEoUG_{lk}^{peak} = \frac{APIDF_{lk}^{peak} \times PG_k^{peak}}{AI_l^{peak}} \tag{22}$$

$$REoUL_{lk}^{peak} = \frac{RPIDF_{lk}^{peak} \times QL_k^{peak}}{AI_l^{peak}} \tag{23}$$

$$REoUG_{lk}^{peak} = \frac{RPIDF_{lk}^{peak} \times QG_k^{peak}}{AI_l^{peak}} \tag{24}$$

where the $peak$ superscript denotes the values at system peak. As the fixed charge will be fixed for the entire year, we define the adapted circuit capacity for the levelized annual circuit cost of the capacity to be

$$ACC_l^{peak} = \frac{I_l^{peak}}{CAP_l} \times CC_l \tag{25}$$

where CC_l is the levelized annual cost of circuit l. Thus, the locational charges to load and generation for active and reactive power are

$$AL_k^{peak} = \sum_{l=1}^{m} AEoUL_{lk}^{peak} \times ACC_l^{peak} \tag{26}$$

$$AG_k^{peak} = \sum_{l=1}^{m} AEoUG_{lk}^{peak} \times ACC_l^{peak} \tag{27}$$

$$RL_k^{peak} = \sum_{l=1}^{m} REoUL_{lk}^{peak} \times ACC_l^{peak} \tag{28}$$

$$RG_k^{peak} = \sum_{l=1}^{m} REoUG_{lk}^{peak} \times ACC_l^{peak}. \tag{29}$$

Relative to the per unit, time differentiated charges, given that the PIDFs are approximately constant, the total charges over the year can differ significantly using a fixed, coincident peak charge. In fact, if an individual load at the coincident peak is greater than the average load for that individual customer over the year, then the charges will be higher. Conversely, if the individual load at coincident peak is less than the average load for that individual customer over the year, the charges will be lower.

3) Nonlocational Charges: As mentioned previously, our extent of use method will not allocate all fixed costs based upon the extent of use. The condition under which locational charges will cover the entire fixed cost of an asset are described below. The remaining fixed cost not recovered by locational charges in the case of time differentiated, per unit charges is

$$RCC^t = \sum_{l=1}^{m} \left[CC_l^t - ACC_l^t\right]$$
$$RCC^t = \sum_{l=1}^{m} CC_l^t \left[1 - \frac{I_l^t}{CAP_l}\right] \tag{30}$$

and these costs will be allocated over all load for the year on a per MWh basis.

The remaining nonlocational costs that must be covered for the fixed, coincident peak locational charge are

$$RCC^{peak} = \sum_{l=1}^{m} \left(CC_l - ACC_l^{peak}\right)$$
$$RCC^{peak} = \sum_{l=1}^{m} CC_l \left(1 - \frac{I_l^{peak}}{CAP_l}\right). \tag{31}$$

and these costs will be allocated based on the individual loads at the coincident peak.

C. When Locational Charges Cover All Fixed Costs of an Asset

In general, our method does not recover all of the fixed costs through locational charges. However, the locational charges defined above can recover all fixed costs when the circuit is fully loaded. Obviously, this results directly from the proposed allocation strategy but can also be easily verified. Let us calculate the total amount recovered by locational charges applied to all busbars, for a given line l, at time t, when the current equals the circuit capacity

$$Loc_l^t = ACC_l^t \times \sum_{k=2}^{n} \left(AEoUL_{lk}^t + AEoUG_{lk}^t + REoUL_{lk}^t + REoUG_{lk}^t\right) \tag{32}$$

$$Loc_l^t = \frac{ACC_l^t}{AI_l^t} \times \sum_{k=2}^{n} \left(APIDF_{lk}^t \times \left(PL_k^t + PG_k^t\right) \times RPIDF_{lk}^t \times \left(QL_k^t + QG_k^t\right)\right) \tag{33}$$

$$Loc_l^t = \frac{I_l^t \times CC_l^t}{CAP_l \times AI_l^t} \times \sum_{k=2}^{n} \left(APIDF_{lk}^t \times \left(PL_k^t + PG_k^t\right) + RPIDF_{lk}^t \times \left(QL_k^t + QG_k^t\right)\right) \tag{34}$$

$$Loc_l^t = \frac{I_l^t}{CAP_l} \times CC_l^t \times \frac{1}{AI_l^t} \times AI_l^t. \tag{35}$$

Then, as $I_l^t = CAP_l$, $Loc_l^t = CC_l^t$.

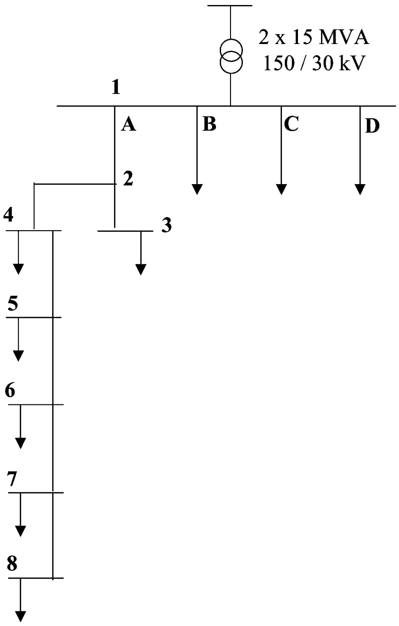

Fig. 1. Rural distribution network.

TABLE I
TYPICAL DATA FOR 120AlAl CONDUCTOR

$r(\Omega/km)$	$x(\Omega/km)$
0.3016	0.3831

TABLE II
INFORMATION DATA FOR THE RURAL RADIAL DISTRIBUTION NETWORK

Sending bus	Receiving bus	Length (km)	Type of Conductor
1	2	10.0	120AlAl
2	3	1.6	120AlAl
2	4	26.0	120AlAl
4	5	3.0	120AlAl
5	6	1.5	120AlAl
6	7	5.6	120AlAl
7	8	13.5	120AlAl

Note that the same can be shown for the fixed, coincident peak charge substituting peak values for time differentiated values and the levelized annual cost for the levelized hourly cost.

IV. APPLICATION-NETWORK CHARACTERISTICS

Let us consider the rural radial distribution network of Fig. 1. The characteristics of the distribution network are meant to reflect conditions in Uruguay, where there are potentially long, radial lines. This network consists of a busbar (1), which is fed by a 150/30 kV transformer and four radial feeders (A, B, C, and D). The network data are shown in Tables I and II. For the purpose of simplicity, we will just consider feeder A for our calculations. Feeder A consists of a 30 kV overhead line feeding six busbars (3, 4, 5, 6, 7, and 8). Except for the case of busbar 4, which is an industrial customer, all the other busbars are 30/15 kV substations providing electricity to low voltage customers (basically residential). In theory, we could apply our tariff scheme to voltages 15 kV and lower, but the cost of metering may be prohibitive at these lower voltages. We will assume then that the industrial customer has the load profile of Fig. 2, and the residential customers have the load profile of Fig. 3. The load profiles used in this section have been taken from a database of the state-owned electric utility in Uruguay. As can be seen in the figures, the residential load profiles follow a typical pattern with daily peaks in the evening. The seasonal peak is in the winter season. The industrial load profile is from a particular customer

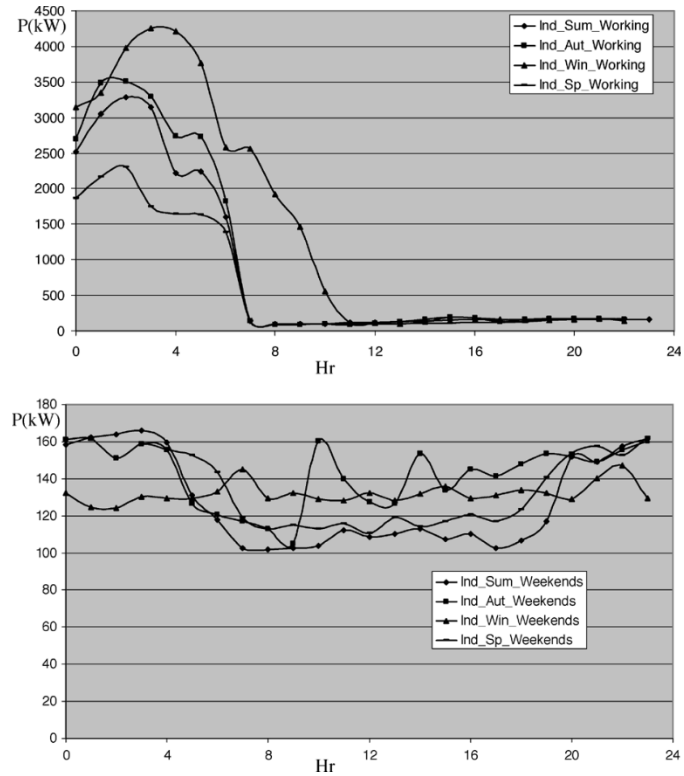

Fig. 2. Daily load profiles for the industrial customer.

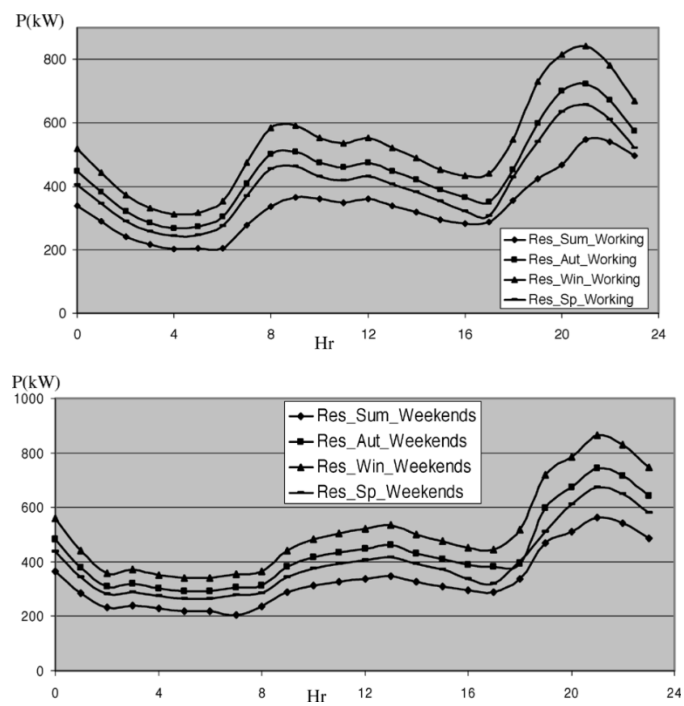

Fig. 3. Daily load profiles for the residential customers.

TABLE III
BENCHMARK: YEARLY CHARGES IN USD USING AVERAGE TARIFF OF 5.40 USD/MWh

Bus	3	4	5	6	7	8
$Charge$	20146	33909	20146	20146	20146	20146

that operates at night due to the tariff structure in Uruguay that encourages usage at night, with daily peaks between midnight and 4 a.m. and a seasonal peak in the winter. For all cases, the power factor for load is assumed to be 0.9 lagging.

We will also run cases with the same distribution network of Fig. 1 but with generator G connected at bus 8. G is a 1 MVA synchronous generator operating at 0.95 lagging power factor. We assume this DG unit runs in all hours along the year at full capacity, except for the weekends when it runs at half capacity. We also assume that G has a cost that is below the system price at these hours for the cases with DG.

As it can be seen, each load profile has eight different scenarios corresponding to seasons and to weekdays and non working days. We will assume that the levelized annual fixed cost of the considered network is USD 134 640, which is reflective of prices in Uruguay.

V. APPLICATION RESULTS

In the case of our network, our benchmark for comparison is a per MWh charge where the fixed cost is averaged over all load for the entire year, which is $5.40/MWh, and the yearly charges for each bus can be seen in Table III. Note that for all of our cases, there is no load at buses 1 and 2; thus, there is no need to report any results for those buses.

Overall, our results show, as expected, residential customers (i.e., same load profiles) locational charges increase with the distance between the customer and the PSP. The more circuits over which power demanded at bus k must travel, the greater is the charge. This reflects the "extent of use" philosophy behind the methodology: the greater the extent of use, the greater the charges will be. The magnitude of the locational charges for each bus will be discussed in more detail below.

We have examined and priced out four cases. Two cases are assessing locational charges on a time differentiated, per unit basis with and without DG, and the other two cases are assessing a fixed, coincident peak locational charge with and without DG. A summary of locational and remaining charges by case can be seen in Table IV. In all cases, the net amount paid to the distribution company should be exactly equal to the fixed cost of $134 640 for the network. However, in the cases with DG, DG receives payments, represented by negative payments, for the "capacity it creates" by locating at bus 8 and generating counterflow that reduces line loading. Moreover, the demand customers, whom we have assumed pay for the network, pay more than the capital cost of the network. The reason is that they are paying for the "extra capacity created" by the DG resource in addition to the actual network capacity. This would be no different than if the distribution company added capacity itself and assessed those charges to demand customers.

With respect to the magnitude of the locational charges in Table IV, there are two things that stand out. The first is that the

TABLE IV
SUMMARY OF LOCATIONAL, REMAINING, AND TOTAL CHARGES BY CASE IN USD/yr

	Bench-mark	Per Unit No DG	Per Unit DG	Fixed No DG	Fixed DG
T_{Loc} Demand	—	24133	20732	51230	46359
T_{Loc} DG	—	—	-4425	—	-4472
T_{Rem}	134640	110507	118333	83410	92717
Tot Demand	134640	134640	139065	134640	139076

TABLE V
DISTRIBUTION NETWORK WITHOUT DG: SUMMARY OF CHARGES IN USD/yr BY BUS

Total locational and remaining charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	3	4	5	6	7	8
T_{Loc}	1047	5855	3641	3783	4297	5510
T_{Rem}	16536	27833	16536	16536	16536	16536
Tot	17583	33688	20177	20319	20833	22046

locational charges for demand are greater without DG in both pricing cases. This is due to the network being more heavily loaded without DG, implying the adapted circuit cost used for allocating locational charges is greater than the cases with DG, thereby leading to the higher charges. The second item that stands out is that the fixed, coincident peak locational charges are greater than the per unit, time differentiated charges. As discussed in Section III, the per unit, time differentiated charges are quite stable over hours and seasons; thus, the total charges in the per unit case are approximately equal to the average load multiplied by the per unit rate multiplied by 8760. However, in the coincident peak case, the load that is determining the yearly charge is the peak, not the average, thus leading to higher overall locational charges.

Below we discuss the various cases and examine more closely the financial impacts at each bus as well as overall properties of those cases.

A. Time Differentiated Per Unit Locational Charges

1) No DG: Computation of the network in this case leads to the results of Table V and Table IX and Figs. 4–7 of Appendix A.

The use of each circuit is due to both active and reactive power flows. For this example, active related charges are approximately 80% of the locational charge, while reactive related locational charges account for the other 20%. Overall, the locational charges recover approximately 18% of the network fixed cost while the other 82% is recovered by the nonlocational charge as seen in Table IX.

Moreover, as discussed in Section III and discussed above, the per unit (MWh or MVArh) charges are relatively stable over hours of the day, weekdays or weekends, and over seasons as can be seen in Figs. 4–7 of Appendix A. We have chosen buses 3, 4, and 8 to show this stability for both residential and industrial loads as well as the fact that location does not affect the stability of the per unit charge. The slight variations that do exist are such

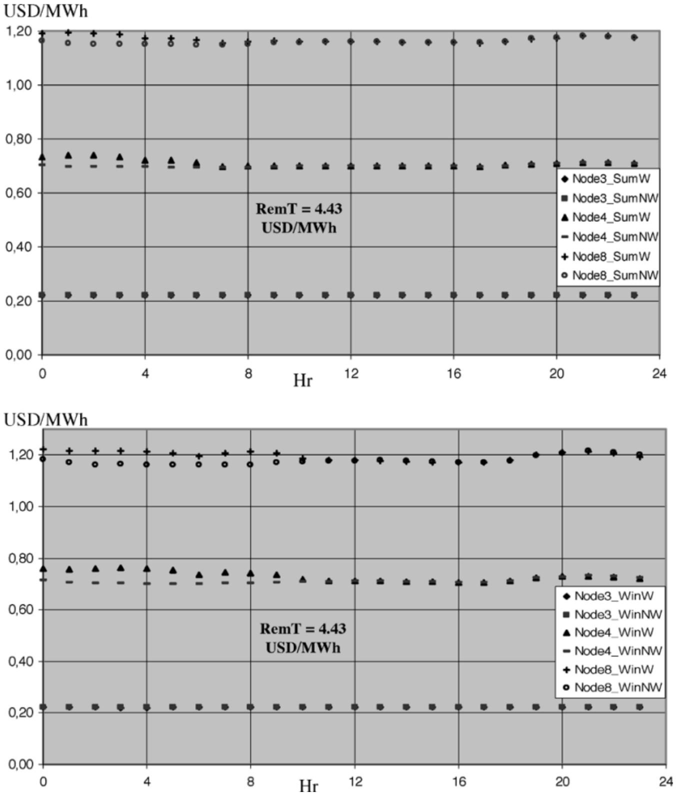

Fig. 4. Active locational tariffs for demand during summer and winter, for working and nonworking days, nodes 3, 4, and 8 (USD/MWh).

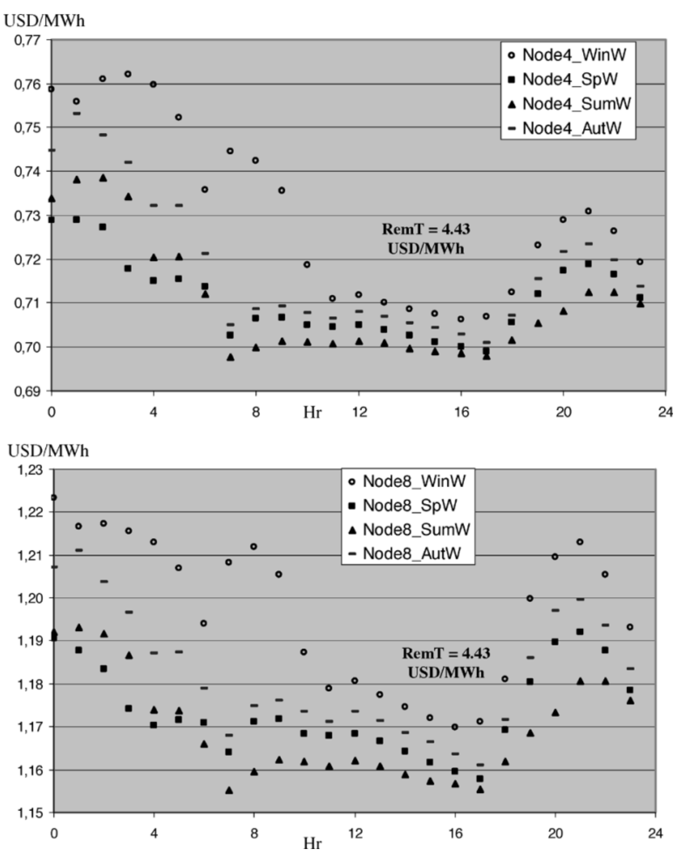

Fig. 5. Active locational tariffs for demand at different seasons, for working days, nodes 4 and 8 (USD/MWh).

that the per unit charge difference are no more that 2.5% of the remaining nonlocational per MWh charge of $4.43/MWh.

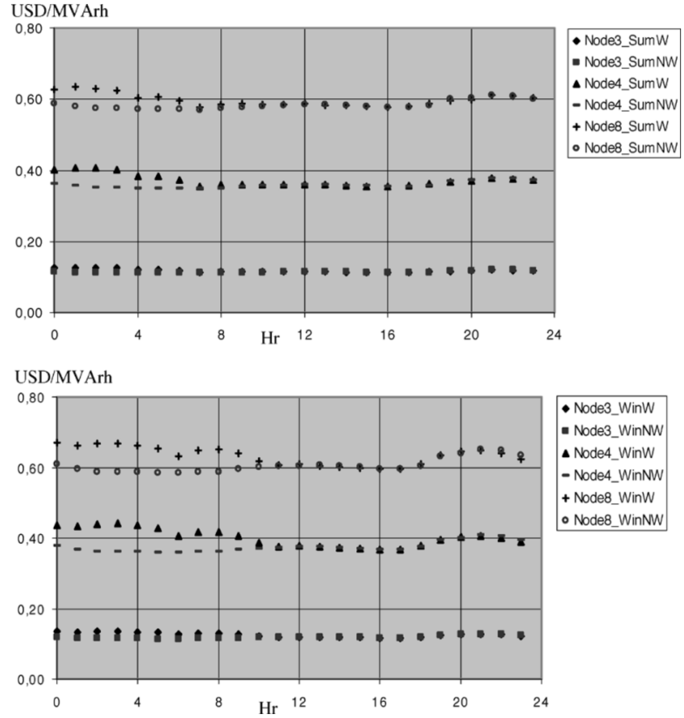

Fig. 6. Reactive locational tariffs for demand during summer and winter, for working and nonworking days, nodes 3, 4, and 8 (USD/MVArh).

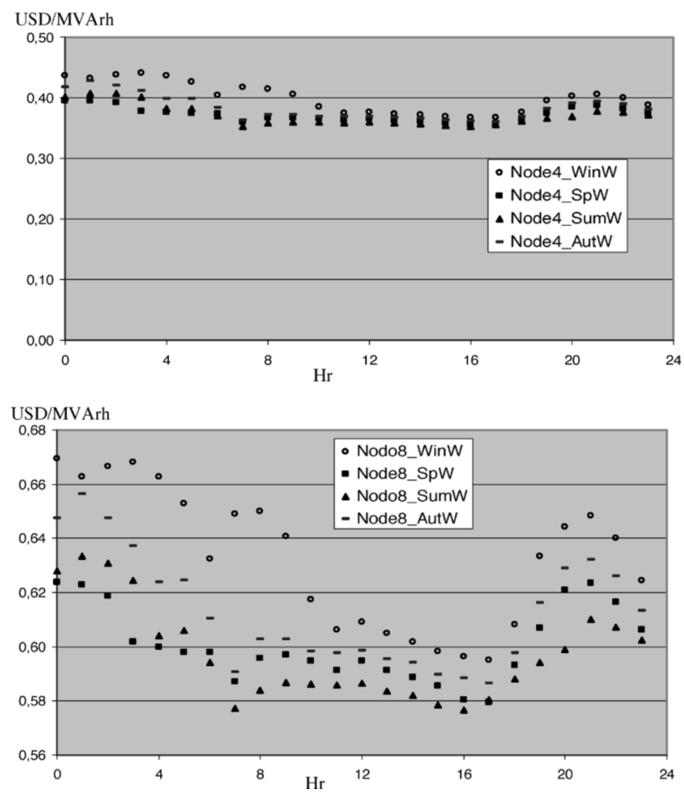

Fig. 7. Reactive locational tariffs for demand at different seasons, for working days, nodes 4 and 8 (USD/MVArh).

Table V summarizes the locational, nonlocational (remaining), and total fixed cost charges by bus for the year. Table IX in Appendix A shows the total active and reactive locational charges for each busbar, in USD/yr for each season. Figs. 4–7 of Appendix A show the per unit charge and its variation over hour and season for buses 3, 4, and 8.

TABLE VI
DISTRIBUTION NETWORK WITH DG: SUMMARY OF CHARGES IN USD/yr BY BUS

Total locational and remaining charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	3	4	5	6	7	$8D$	$8G$
T_{Loc}	1033	5704	3535	3648	3809	3003	-4425
T_{Rem}	17706	29801	17706	17706	17706	17706	-
Tot	18739	35505	21241	21354	21515	20709	-4425

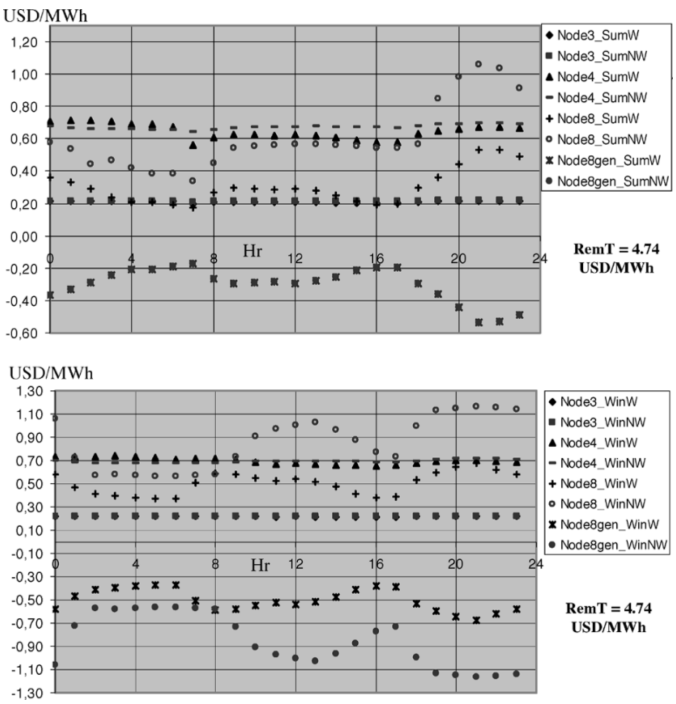

Fig. 8. Active locational tariffs for demand and generation during summer and winter, for working and nonworking days, nodes 3, 4, and 8 (USD/MWh).

The financial implications of locational fixed charges is revealing as well from Table V. Now consider the residential customer at bus 3. Under our proposed methodology and time differentiated per unit charge, the total charges for the year are \$17 538 versus benchmark charges of \$20 146, a 13% savings, due to the fact that load at bus 3 does not affect the rest of the network or affects it very little. The residential customer at the end of the line at bus 8, however, pays more: total charges of \$22 046 versus the benchmark of \$20 146, a 9.5% increase. Again, this is as expected as the customer at bus 8 affects all the assets in the system. As for the industrial customer at bus 4, its charge change very little in this case \$33 688 versus the benchmark of \$33 909.

2) With DG: Computation of the network in this case leads to the results of Table VI and Table X and Figs. 8–11 of Appendix A.

For this example, active related locational charges are approximately 76% of the locational charge inclusive of payments to DG, while reactive related locational charges account for the other 24% as seen in Table X in Appendix A. Overall, the locational charges, inclusive of payments to DG, recover approximately only 12% of the network fixed cost while the other 88% is recovered by the nonlocational charge as seen in Table X in Appendix A.

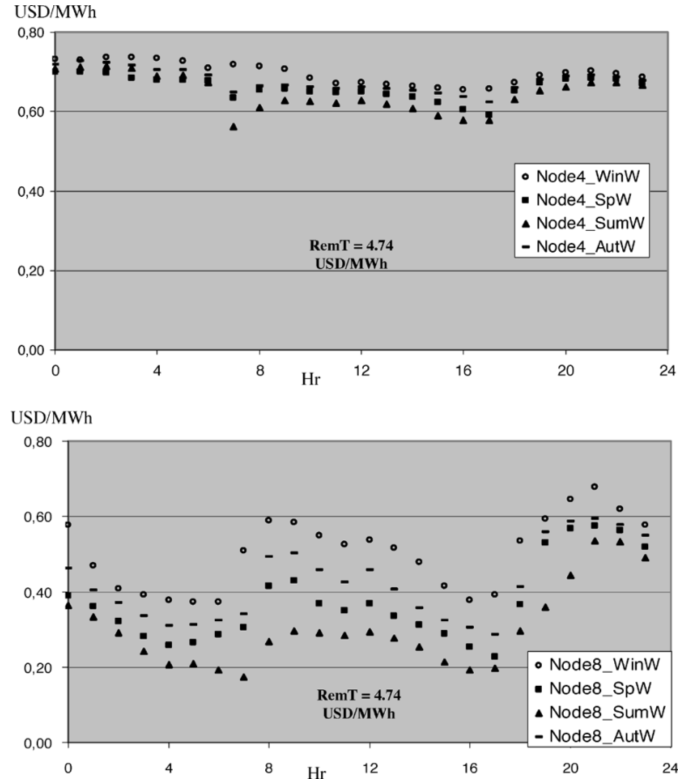

Fig. 9. Active locational tariffs for demand and generation at different seasons, for working days, nodes 4 and 8 (USD/MWh).

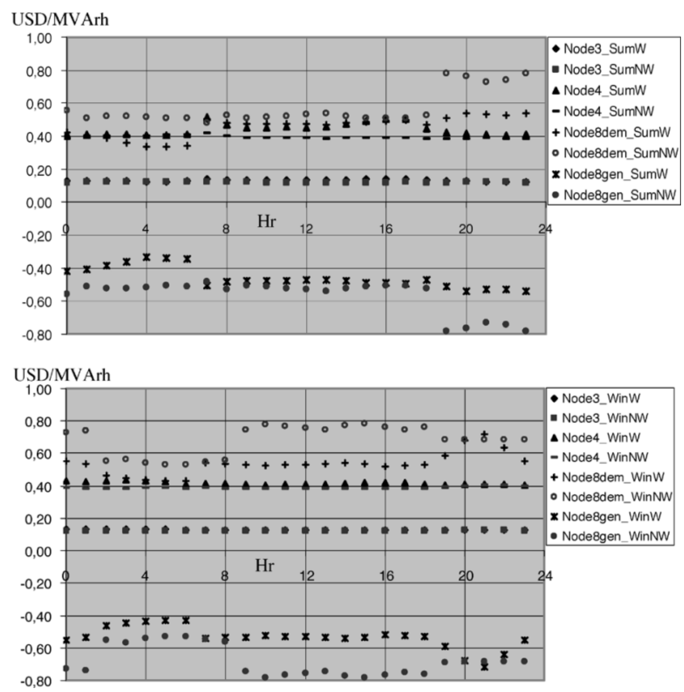

Fig. 10. Reactive locational tariffs for demand and generation during summer and winter, for working and nonworking days, nodes 3, 4, and 8 (USD/MVArh).

In this case, charges (both active and reactive related charges) for generator G are negative, reflecting the counterflow that the DG resource is providing to free up circuit capacity. Another way of viewing this result, as stated previously, is that the negative charges are really payments to the DG for "creating" extra capacity in the network. In addition, the payments made to the generator are greater at times of greater network utilization, such

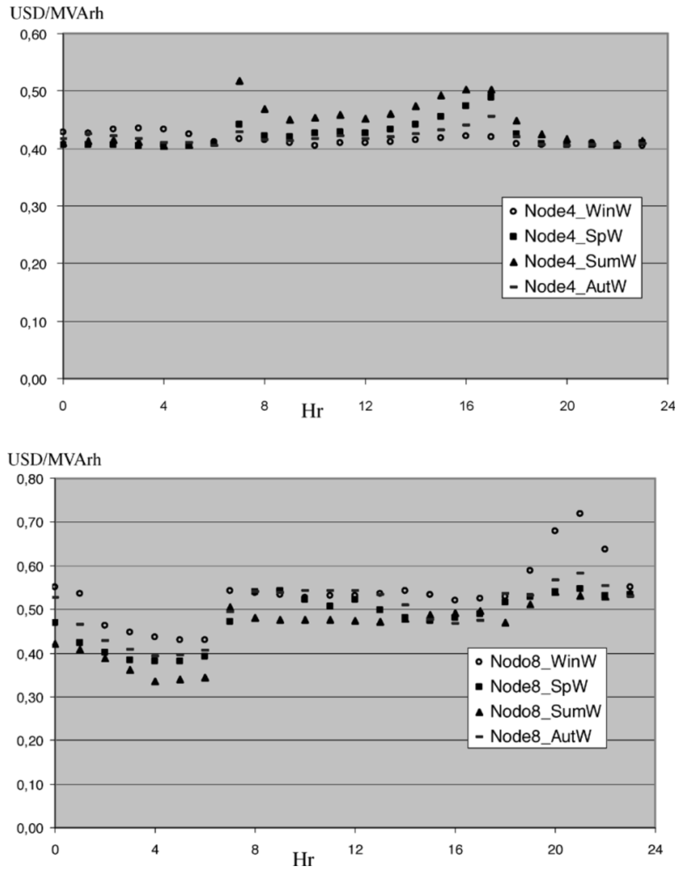

Fig. 11. Reactive locational tariffs for demand and generation at different seasons, for working days, nodes 4 and 8 (USD/MVArh).

as the winter season and at greater loading attributable to residential loads at their peak hours at buses 5–8, reflecting the increased value the DG resource provides as the network becomes more heavily loaded as shown in Figs. 8–11 of Appendix A.

Overall, the presence of DG also alters the tariffs of demand customers. Overall, locational charges for load decrease relative to the case without the DG resource but only by about 14% of the locational charges without DG, and by bus, the decrease is greater the closer the load is to the DG resource. This reduced locational charge is attributed to the decreased line loading from the counterflow from the DG resource.[3] Note, for instance, that there is a large reduction in locational charges for the demand at bus 8. Due to the reduced line loading, the nonlocational charge increases from $4.43 to $4.74/MWh or by 7% over the case without DG.

The overall network capital charge will increase for load customers on the network, as mentioned above. This result should not be surprising as load customers are benefiting from, and paying for, the virtual increase in network capacity created by the DG resource. However, the total cost to load customer may decrease with the decrease in line losses induced by the increased network capacity, though we do not examine losses here. In any event, the total charges paid by load, relative to the benchmark are all higher, except for bus 3, and they are all higher than the case without DG, except for bus 8, which benefits directly from being at the same bus as DG.

[3]Our extent of use factors are weighted by a linear approximation of the current flow, which for the value of any withdrawal, is less than the actual current as current is a concave (square root) function of withdrawals. Going back to (12) and (13), with the reduction in line loading, actual current flow decreases by more than the linear approximation resulting in lower charges for the same load.

TABLE VII
DISTRIBUTION NETWORK WITHOUT DG: SUMMARY OF PEAK CHARGES IN USD/yr

Total locational and remaining charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	3	4	5	6	7	8	TOT
Tot_{Loc}	827	36230	3039	3145	3535	4455	51230
Tot_{Rem}	4675	60035	4675	4675	4675	4675	83410
Tot	5502	96265	7714	7820	8210	9130	134640

TABLE VIII
DISTRIBUTION NETWORK WITH DG: SUMMARY OF PEAK CHARGES IN USD/yr

Total locational and remaining charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	3	4	5	6	7	$8D$	$8G$
Tot_{Loc}	819	35200	2940	3004	2668	1764	-4472
Tot_{Rem}	5196	66737	5196	5196	5196	5196	-
Tot	6015	101937	8136	8200	7864	6960	-4472

B. Fixed, Coincident Peak Locational Charges

1) No DG: A summary of the fixed, coincident peak locational charges without DG can be found in Table VII and Table XI in Appendix A. As discussed above, the total charges paid, relative to the time differentiated per unit charges, will depend on whether the load at the coincident peak is less than or greater than the average load over the year. For example, the loads at all residential (3, 5, 6, 7, 8) buses pay lower locational charges, and lower overall charges, than they did under the other pricing regime because their load at the peak hour is less than the average load over the year. The overall charges for residential loads are also much lower than the benchmark charges. In fact, the coincident peak occurs in hour 3 during the winter season and is driven by the industrial customer at bus 4. Moreover, if one is to examine the load profiles in Figs. 2 and 3, it is easy to see that at the peak hour, residential customers are close to their minimums rather than their peaks. This result is purely an artifact of the data we have on loads in Uruguay. If the residentials peaked at about the same time as the industrial customer, they too would pay more than under the per unit charges, just as the industrial customer at bus 4 does. The industrial customer, because it is driving the peak, pays more than six times more in locational charges than it did under the other pricing mechanism and drives the overall more than doubling in locational charges.

2) With DG: The results with distributed generation can be seen in Table VIII. Much like the time differentiated, per unit pricing scheme with DG, DG leads to an overall decrease of 10% in locational charges for loads, and that decrease is greater for buses closer to the DG resource (see Tables VIII and XI). Moreover, the overall network capital charge will increase, as it did in the previous pricing scheme, for load customers on the network. Again, load customers are benefiting from, and paying for, the virtual increase in network capacity created by the DG resource. It is interesting to note that the DG resources revenues from creating extra capacity have changed little, increasing by just over 1%. For loads, the overall charges have increased versus fixed charges without DG, except for loads at buses 7 and 8, which benefit greatly from DG at the peak. Also, just as before with fixed charges without DG, the residential buses pay far less than the benchmark and far less than under the per unit prices.

VI. Conclusion

This paper has presented a new methodology for the allocation of fixed costs at the MV distribution level. The methodology, based on the widely used MW-mile for transmission networks, uses power to current distribution factors in order to measure the extent of use imposed by customers to the network and thus can be referred as the "Amp-mile" or "I-mile" method for distribution networks. Unlike traditional tariff designs that average fixed costs over all load, our methodology uses cost causality (extent of use) to assign part of the fixed costs of the network. In particular, DG is paid for the reduction of network utilization (a virtual increase in network capacity). Moreover, demand customers who impose a low network use have, within the proposed methodology, lower charges than those that impose a high network use. The price signals sent with the Amp-mile method become stronger as network utilization increases. In particular, if the network were fully loaded, all fixed costs would be recovered by the locational charges.

Applying our methodology to a distribution network that has characteristics found in Uruguay, and for two different pricing schemes, we show the financial incentives are in the desired direction, and the signals are strongest for those loads that drive the coincident peak of the system and that are far away from the power supply point. Moreover, using a fixed, coincident peak charge recovers more of the fixed costs through locational charges than does a time differentiated, per unit charge. Finally, we find that time differentiating the per unit charge does not aid in pricing for cost causality as the per unit charge is stable over hours of the day, days of the week, and seasons.

Appendix A
Application: Results

Tables IX and X show the active and reactive per MWh locational charges by season, as discussed in Section V, subsection A in the body of this paper. Mapping these out for selected nodes in Figs. 4–11 for both active and reactive power, we can see how these per MWh locational charges vary throughout the day and vary by season.

Tables XI shows the peak coincident locational charges for both active and reactive power with and without distributed generation, as discussed in Section V, subsection B in the body of this paper.

Appendix B
Power Flow and Analytical Derivatives Calculation

The equations for the power flow are

$$i(k)=\sum_{h\in H_k^{in}} f(h)-\sum_{h\in H_k^{out}} f(h), \quad \forall k\in V \tag{36}$$

$$v(k)conj\left(i(k)\right)=s(k)=p(k)+jq(k), \quad \forall k\in V \tag{37}$$

$$v(k_{h,ini})-v(k_{h,end})=(r(h)+jx(h))\,f(h), \quad \forall h\in E \tag{38}$$

TABLE IX
Distribution Network Without DG: Charges in USD/yr

Active locational charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	Sum_L	Aut_L	Win_L	Sp_L	Tot_Loc	$RemT$
3	162	217	254	196	829	16536
4	1020	1149	1701	753	4623	27833
5	562	757	899	682	2900	16536
6	584	787	934	708	3013	16536
7	665	895	1063	805	3428	16536
8	856	1151	1363	1037	4407	16536

Reactive locational charges for demand, all seasons, for working days and weekends (USD/yr)

Bus	Sum_L	Aut_L	Win_L	Sp_L	$Total$
3	42	57	69	50	218
4	266	306	466	194	1232
5	141	194	235	171	741
6	147	201	244	178	770
7	165	227	275	202	869
8	211	288	347	257	1103

Remaining amount, all seasons, for working days and weekends(USD/yr)

Sum_L	Aut_L	Win_L	Sp_L	$Total$
28839	27431	25810	28427	110507

TABLE X
Distribution Network With DG: Charges in USD/yr

Active locational charges for demand and generation, all seasons, for working days and weekends(USD/yr)

Bus	Sum_L	Aut_L	Win_L	Sp_L	Tot_Loc	$RemT$
3	156	211	249	190	806	17706
4	973	1105	1641	717	4436	29801
5	511	716	860	637	2724	17706
6	519	738	889	653	2799	17706
7	492	754	946	649	2841	17706
8-dem	310	532	728	438	2008	17706
8-gen	-626	-844	-999	-754	-3223	-

Reactive locational charges for demand and generation, for working days and weekends (USD/yr)

Bus	Sum_L	Aut_L	Win_L	Sp_L	$Total$
3	45	59	70	53	227
4	282	314	465	207	1268
5	165	210	244	192	811
6	172	220	256	201	849
7	188	254	298	228	968
8-dem	181	260	328	226	995
8-gen	-279	-304	-327	-292	-1202

Remaining amount, all seasons, for working days and weekends(USD/yr)

Sum_L	Aut_L	Win_L	Sp_L	$Total$
30571	29435	28012	30315	118333

where

- $i(k)$ complex charging current for node k;
- $f(h)$ complex current flowing through line h;
- $v(k)$ complex voltage at node k;
- $conj(z)$ conjugate of complex number z;
- $s(k)$ loading apparent power at node k, being $p(k)$, $q(k)$, the active and reactive power, respectively; $p(k), q(k) > 0$ corresponds to consumption/demand, $p(k), q(k) < 0$ corresponds to generation;
- $r(h)$, $x(h)$ resistance and the reactance for line h;
- H_k^{in}, H_k^{out} sets of entry lines and salient lines for node k, respectively;

TABLE XI
FIXED COINCIDENT PEAK CHARGES USD/yr

Active related charges (P), reactive charges (Q), and remaining charges (R), for cases with and without DG

Bus	P_{noDG}	Q_{noDG}	R_{noDG}	P_{DG}	Q_{DG}	R_{DG}
3	638	189	4675	632	187	5196
4	28305	7925	60035	27371	7829	66737
5	2377	662	4675	2267	673	5196
6	2462	683	4675	2288	716	5196
7	2775	760	4675	1944	724	5196
8-d	3515	940	4675	1134	629	5196
8-g	-	-	-	-3254	-1218	-
Total Load	40072	11159	83410	35636	10758	92717

V set of nodes;
E set of lines.

Equation (36) corresponds to the current balance at each node, (37) is the definition of the apparent power for each node relating voltage, current, and power, and (38) is Ohms law applied to each line. Note that all magnitudes are in per unit.

For the case we are studying, our unknown variables are v and i, while the known variables are all ps and qs. The only exception to this is the voltage at the slack bus, which is known and set at 1 p.u.

We will work with the matricial form of (36)–(38)

$$i = A^T f \tag{39}$$
$$v.* conj(i) = p + jq \tag{40}$$
$$Av = -(r + jx).* f \tag{41}$$

where A is the incident matrix lines-nodes defined as follows:

$$\begin{aligned} &A/ \\ &A(h, k_{h,end}) = 1 \\ &A(h, k_{h,ini}) = -1 \\ &A(h,k) = 0, \qquad \forall k \neq k_{h,ini}, k_{h,end} \end{aligned} \tag{42}$$

The notation $.*$ indicates the operation element by element.

For our particular case, where the network is radial we have $n_{nod} = n_{lines} + 1$, and the slack bus k_s is the PSP, where the distribution network connects to the transmission network.

Let us call V_{ns} the set of nodes different from the slack bus; then, $V = \{k_s\} \cup V_{ns}$. We will use a similar notation for vectors v, i and for matrix A

$$v = (v_s, v_{ns}), \quad i = (i_s, i_{ns}), \quad A = (A_s, A_{ns})$$

where $v_s = v_0$ is known, A_s is the column k_s of A, and A_{ns} is a square matrix obtained from withdrawing the column k_s of A. It is possible to prove that A_{ns} is invertible; we are not going to do so here.

Then (39)–(41) can be written as follows:

$$i_s = A_s^T f \tag{43}$$
$$i_{ns} = A_{ns}^T f \tag{44}$$
$$v_0 conj(i_s) = p_s + jq_s \tag{45}$$
$$v_{ns}.* conj(i_{ns}) = p_{ns} + jq_{ns} \tag{46}$$
$$A_s v_0 + A_{ns} v_{ns} = -Rf \tag{47}$$

where R is a diagonal matrix with vector $r + jx$ at the diagonal. In order to find v_{ns}, i_{ns}, f, we can focus in the resolution of (44), (46), and (47). Afterwards, (43) and (45) allow us to calculate the current and the power at the slack bus once fluxes f through the lines are known. Let us call

$$A_2 = \left(A_{ns}^T\right)^{-1}.$$

We can then calculate f from (44), obtaining

$$f = A_2 i_{ns}. \tag{48}$$

Then, substituting in (47), we have

$$A_s v_0 + A_{ns} v_{ns} = -RA_2 i_{ns}$$

and then

$$\begin{aligned} v_{ns} &= A_{ns}^{-1}(-A_s v_0 - RA_2 i_{ns}) \\ v_{ns} &= -v_0 A_2^T A_s - A_2^T RA_2 i_{ns} \\ v_{ns} &= d + Di_{ns} \end{aligned} \tag{49}$$

where $d = -v_0 A_2^T A_s$ is a column vector of n_{line} elements, and $D = -A_2^T RA_2$ is a square matrix of size n_{line}.

To sum up, we have to solve a nonlinear system of equations consisting in (46) and (49), which may be written as

$$i_{ns} = \frac{(p_{ns} - jq_{ns})}{conj(v_{ns})} \tag{50}$$
$$v_{ns} = d + Di_{ns}. \tag{51}$$

The advantage of this reasoning is that allows to calculate the currents from the voltages and viceversa in a form that is adequate to an iterative algorithm.

A. Iterative Algorithm

The iterative algorithm used is as follows:

First Step: Choose tolerance ε and set $v(k) = v_0 \ \forall k \in V_{ns}$.

Iterative step:

1) Save in v_{old} the actual value of voltage vector v_{ns}.
2) Calculate the current vector i_{ns} using (50).
3) Calculate the voltage vector v_{ns} using (51).
4) If $\|v_{ns} - v_{old}\| < \varepsilon$, the iteration is finished. Otherwise, go to 1).

Final step: Calculate f using (48), then i_s using (43), and then active and reactive powers p_s, q_s using (45).

The convergence of the method can be proven in a similar way as it is done in [21]. A linear convergence, corresponding to the limit: $\lim_{iter\to\infty} \|v^{iter+1} - v*\|/\|v^{iter} - v*\| < \beta$, with $\beta < 10^{-2}$, can be proven.

In practice, a fast convergence, reaching a tolerance of 10^{-6} in vector v within an average of six iterations, can be observed.

B. Derivatives Calculation

1) Derivatives of Node Currents With Respect to Loading Active and Reactive Powers: From (46) and (51), which relate current, voltage, and active and reactive powers at network nodes

$$\begin{aligned} conj(i_{ns}).* v_{ns} &= p_{ns} + jq_{ns} \\ v_{ns} &= d + Di_{ns} \end{aligned}$$

we obtain the node loading power as a function of the node loading current

$$s_{ns} = p_{ns} + jq_{ns} = F(i_{ns}) = conj(i_{ns}).*(d + Di_{ns}). \tag{52}$$

The idea is to find the matrix derivatives of powers with respect to currents and then calculate the inverse.

To do this, we first make a distinction between the real and imaginary parts of the complex magnitudes

$$i_{ns} = z + jy, \quad D = D_1 + jD_2.$$

Then, substituting in (52), we obtain two real functions

$$\begin{aligned} p_{ns} &= F_1(z,y) \\ q_{ns} &= F_2(z,y) \\ p_{ns} &= z.*(d + D_1 z - D_2 y) \\ &\quad + y.*(D_2 z + D_1 y) \qquad (53) \\ q_{ns} &= -y.*(d + D_1 z - D_2 y) \\ &\quad + z.*(D_2 z + D_1 y). \qquad (54) \end{aligned}$$

In order to find the matrix of partial derivatives, we will see at first what the Jacobian matrix $\partial f/\partial x$ of a vectorial function $f : R^N \to R^N$ defined as $f(x) = u(x).*v(x)$ looks like.

As $f_k(x) = u_k(x)v_k(x)$, $\partial f_k(x)/\partial x_h = (\partial u_k(x)/\partial x_h)v_k(x) + u_k(x)(\partial v_k(x)/\partial x_h)$. Then row k of $\partial f/\partial x$ matrix is

$$\frac{\partial f_k}{\partial x} = v_k \frac{\partial u_k}{\partial x} + u_k \frac{\partial v_k}{\partial x}$$

and then

$$\frac{\partial f}{\partial x} = diag(v)\frac{\partial u}{\partial x} + diag(u)\frac{\partial v}{\partial x}. \qquad (55)$$

As a result, applying (55) to our functions in (53) and (54), we have

$$\begin{aligned} \frac{\partial F_1}{\partial z} &= diag(z)D_1 + diag(y)D_2 \\ &\quad + diag(d + D_1 z - D_2 y) \qquad (56) \\ \frac{\partial F_1}{\partial y} &= -diag(z)D_2 + diag(y)D_1 \\ &\quad + diag(D_2 z + D_1 y) \qquad (57) \\ \frac{\partial F_2}{\partial z} &= diag(z)D_2 - diag(y)D_1 \\ &\quad + diag(D_1 z + D_2 y) \qquad (58) \\ \frac{\partial F_2}{\partial y} &= diag(z)D_1 + diag(y)D_2 \\ &\quad - diag(d + D_1 z - D_2 y). \qquad (59) \end{aligned}$$

The desired Jacobian matrices are then

$$J_0 = \frac{\partial(p_{nr}, q_{nr})}{\partial(z,y)} = \begin{pmatrix} \frac{\partial F_1}{\partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial z} & \frac{\partial F_2}{\partial y} \end{pmatrix}$$

and

$$J_1 = \frac{\partial(z,y)}{\partial(p_{nr}, q_{nr})} = \begin{pmatrix} \frac{\partial F_1}{\partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial z} & \frac{\partial F_2}{\partial y} \end{pmatrix}^{-1}. \qquad (60)$$

2) Derivatives of the Line Currents With Respect to Node Currents: From (48), and including notation $f = f_1 + jf_2$, we have that $f_1 + jf_2 = (A_{ns}^T)^{-1}(z + jy)$, and then

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_2}{\partial y} = \left(A_{ns}^T\right)^{-1}, \quad \frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial z} = 0.$$

Finally, the Jacobian matrix is

$$J_2 = \begin{pmatrix} \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} \left(A_{ns}^T\right)^{-1} & 0 \\ 0 & \left(A_{ns}^T\right)^{-1} \end{pmatrix}. \qquad (61)$$

3) Derivatives of Absolute Values of Line Currents With Respect to Node Active and Reactive Powers: We would like to calculate the Jacobian matrix $J_6 = \partial I/\partial(p_{ns}, q_{ns})$ with the partial derivatives of absolute values $I(h) = abs(f(h)) = \sqrt{f_1(h)^2 + f_2(h)^2}$ of the line currents with respect to the active and reactive powers at nodes (except the slack).

We have already calculated matrix $J_2 = \partial(f_1, f_2)/\partial(z,y)$ with the derivatives of the line currents with respect to node currents $i_{ns} = z + jy$ and matrix $J_1 = \partial(z,y)/\partial(p_{ns}, q_{ns})$ with the derivatives of node currents with respect to active and reactive powers.

Then, the Jacobian matrix we are looking for now can be calculated as

$$J_6 = \frac{\partial I}{\partial(p_{ns}, q_{ns})} = \frac{\partial I}{\partial(f_1, f_2)} \frac{\partial(f_1, f_2)}{\partial(p_{ns}, q_{ns})} = J_7 J_{21} \qquad (62)$$

with

$$J_{21} = \frac{\partial(f_1, f_2)}{\partial(p_{ns}, q_{ns})} = J_2 J_1$$

and

$$J_7 = \frac{\partial I}{\partial(f_1, f_2)} = \frac{(diag(f_1) \quad diag(f_2))}{I}.$$

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